MANUEL PABLO RAMIREZ

MODELING AND OPTIMIZATION OF A REAL VEHICLE ROUTING PROBLEM FOR SUSTAINABLE COLLECTION OF USED LUBRICATING OIL

São Paulo
2021

MANUEL PABLO RAMIREZ

MODELING AND OPTIMIZATION OF A REAL VEHICLE ROUTING PROBLEM FOR SUSTAINABLE COLLECTION OF USED LUBRICATING OIL

Graduation thesis for Escola Politécnica da Universidade de São Paulo to obtain a degree in Industrial Engineering

São Paulo
2021

MANUEL PABLO RAMIREZ

MODELING AND OPTIMIZATION OF A REAL VEHICLE ROUTING PROBLEM FOR SUSTAINABLE COLLECTION OF USED LUBRICATING OIL

Graduation thesis for Escola Politécnica da Universidade de São Paulo to obtain a degree in Industrial Engineering

Supervisor:

Leonardo Junqueira

Co-supervisors:

Marco Melacini
Lorenzo Prataviera

São Paulo
2021

Catalogação-na-publicação

Ramirez, Manuel Pablo
Modeling and optimization of a real Vehicle Routing Problem for sustainable collection of used lubricating oil / M. P. Ramirez -- São Paulo, 2021.
87 p.

Trabalho de Formatura - Escola Politécnica da Universidade de São Paulo. Departamento de Engenharia de Produção.

1.Roteirização 2.Pesquisa operacional 3.Programação matemática 4.Logística reversa 5.Economia circular I.Universidade de São Paulo. Escola Politécnica. Departamento de Engenharia de Produção II.t.

To my family and friends, who I love so much

ACKNOWLEDGEMENTS

I would like to thank everyone who supported me and believed in me during my journey in Brazil and in Italy.

I thank my supervisor and co-supervisors Leonardo Junqueira, Marco Melacini and Lorenzo Prataviera, who helped me immensely to develop this project and grow as a student, a professional and a person. I also thank all my professors, both from university and school, who have believed in me and encouraged me to always pursuit the best.

I thank my mother Sandra, my father Gustavo, my sister Paloma and all of my family from Argentina, who have always been there for me and taught me so many things. I cannot express enough how much their support and their love were essential for me to become who I am.

Finally, I would like to thank all of my friends. My history up until now has been full of wonderful people who have been by my side through tough and joyful moments. I have met amazing people in a lot of different times and places in my life and I hope to have impacted their life as much as they impacted mine. I would not have been able to achieve anything without them.

Thank you all.

ABSTRACT

The application of circular economy practices in all sorts of processes is a matter that has been receiving increasing attention over the years. One of the well-known applications of those practices occurs in the collection and recycling of used lubricating oil. After being utilized in industrial machinery, lubricants become used oil, which can be upcycled back to base oil and then transformed again into lubricants. This thesis has been developed in a partnership with the largest company in Brazil in the market of collection and recycling of used oil. Since that company uses its own fleet of trucks to attend its clients, it is also in charge of devising a daily routing plan for the collection staff. This thesis is concerned with studying that routing process for a particular business unit in Osasco, São Paulo. The main objectives are to apply the concepts of the class of Vehicle Routing Problems to that case study, to develop a mathematical model that represents the problem at hand and to implement and test that model in a computational program, generating useful insights for the company from the results. The class of Vehicle Routing Problems, due to its complexity and flexibility, is an area of Operations Research that has been of interest for academics and practitioners for over 60 years. The present thesis performs a literature review on some variants of the problem which are related to the problem at hand and a more detailed explanation of the used oil collection process and its context. After that, the case study of the thesis is presented, along with the problem definition. The problem consists in minimizing the fuel consumption for a specific segment of the company's truck fleet, which is in charge of collecting used oil from clients in the Metropolitan Region of São Paulo. The problem presents a series of characteristics, such as access time windows restrictions for collection, a heterogeneous fixed fleet and the possibility for the vehicles to perform multiple trips in the same day. Once the problem is defined, a mathematical model based on the concepts of the Vehicle Routing Problems class is designed to describe it. Given the unique combination of restrictions, the mathematical modeling included additional challenges. One of the main contributions of this thesis is to develop an original way of combining the restrictions of multiple trips for each truck and the access time windows restrictions of the Traffic Restricted Region of São Paulo. The model was then implemented and tested using Python and Gurobi, a solver that applies an exact approach in its algorithm, to reach the best solution for each instance within practically feasible computational times. A first set of tests was used to evaluate the model's performance by combining different parameters of the problem. A second set of tests was performed with real instances from the partner company and its results were compared with the routing software the company currently uses. Results show that the model performs well for instances with a moderate amount of clients, outputting better solutions in terms of truck fuel consumption when compared to the company's software. A recommendation deriving from the results would be for the company to incorporate the model in its routing plans, in order to get potentially better solutions than those it currently uses.

Keywords: Used Oil Collection, Circular Economy, Operations Research, Rich Vehicle Routing Problems.

LIST OF FIGURES

LIST OF TABLES

LIST OF ACRONYMS

CEAP	Circular Economy Action Plan
CVRP	Capacitated Vehicle Routing Problem
CVRS	Commercial Vehicle Routing Software
HVRP	Heterogeneous Fleet Vehicle Routing Problem
ICT	Information and Communications Technology
LNS	Large Neighborhood Search
LP	Linear Programming
MILP	Mixed-Integer Linear Programming
MRSP	Metropolitan Region of São Paulo
MTVRP	Vehicle Routing Problem with Multi-Trips
OECD	Organisation for Economic Co-operation and Development
OR	Operations Research
PPC	Problem Physical Characteristics
RVRP	Rich Vehicle Routing Problem
SC	Scenario Characteristics
SDVRP	Site-Dependent Vehicle Routing Problem
SR	SimpliRoute
TRR	Traffic Restricted Region
TS	Tabu Search
TSP	Travelling Salesman Problem
VRP	Vehicle Routing Problem
VRPATW	Vehicle Routing Problem with Access Time Windows
VRPTW	Vehicle Routing Problem with Time Windows

CONTENTS

1 INTRODUCTION

Transitioning towards a more sustainable model of economic development has been the focus of many governmental policies in recent years. The Brundtland Commission defines sustainability as the “development that meets the needs of the present without compromising the ability of future generations to meet their own needs” (BRUNDTLAND, 1987). One of the most promising ways to foster sustainable development, according to many scholars, is to implement the concepts of circular economy in different production systems, which could be defined as “an industrial economy that is restorative or regenerative by intention and design” (FOUNDATION, 2013). Although circular economy is far from being completely adopted in all processes, some industries have been successful in using this approach, one of them being the lubricants and the used oil industry (PINHEIRO; QUINA; GANDO-FERREIRA, 2020).

Lubricants are an essential part of almost every industrial process nowadays. According to Lubemonitrix (2021), a company specialized in oil analysis tools, the primary function of a lubricant is to prevent friction between two surfaces. It also plays the role of dissipating heat, transporting contaminants, protecting from oxidation and corrosion and transmitting power.

After being used or contaminated, lubricants are highly toxic, presenting danger not only to human health but also to the environment. The substance that remains after using the lubricant is called used lubricating oil or waste oil. According to Brazilian norm NBR1004 (ABNT, 2021), used oil falls in the category of “dangerous waste class I”, which is characterized by either presenting risk to public health, to the environment or both. The reason for it to be in that category is that it has components which present that risk, such as polycyclic aromatic hydrocarbons (PAH), organic acids and heavy metals.

Therefore, companies are advised to discard and recycle it properly, although not all countries enforce that legally. In Brazil, the law that establishes the norms on how to deal with used oil is Resolution nº362/2005 from CONAMA 362 (CONAMA, 2021). The institution responsible for the regulation and supervision of the used oil market is ANP (the acronym stands for "Agência Nacional do Petróleo, Gás Natural e Biocombustíveis", a regulatory governmental institution) (ANP, 2021).

After using the lubricant, the company is visited by a collector, which transports the used oil to a specialized facility in order to recycle it. There it can be refined again or put directly into use by authorized companies. In 2019, more than 480 million liters of used oil were collected for recycling in Brazil (ANP, 2021). However, that figure, although meaningful, can still be improved, given that it represents only 40% of all commercialized oil in the country (MMA, 2020). Countries such as Austria, Netherlands and Belgium have higher rates of collection compared to consumption and could serve as a model to develop the collection network (STAHL; MERZ, 2020).

Table 1 compares the collection rate of used oil in 2019 based on total commercialization, for Brazil, and based on total consumption, for other countries. Although those measures are not exactly the same, they have similar approaches to calculate the rate, which validates their comparison.

Table 1: Collection rates of used oil based on commercialization (Brazil) or consumption (other countries) in 2019

Country	**Collection Rate (%)**
France	34
Spain	38
Brazil	**40**
Italy	42
Germany	43
Austria	47
Netherlands	47
Belgium	58

Source: Stahl and Merz (2020), MMA (2020)

However, not all of the lubricant is correctly recycled. There are clandestine operations which illegally buy used lubricating oil, usually from smaller companies, in order to recycle it and resell it without following the necessary protocols. That can result in misuse of the product and the loss of track of the oil, which could later be incorrectly discarded in the environment. In December 2019, a police operation called "Petrolato" was activated to investigate and apprehend those clandestine operations in 10 Brazilian states. For example, in Rio Grande do Sul, one of those states, more than 13 million liters of lubricant were incorrectly discarded during the first semester of 2019 (MPRS, 2019).

Considering the presented information, it is evident that the Brazilian used oil market presents potential not only for growth towards an enlarged and more efficient oil recycling activity, as it is possible to infer from the comparison to other countries, but also towards a more regularized one.

This graduation thesis aims at applying the concepts from Operations Research to the logistics of a company inserted in the aforementioned scenario, in order to improve its operations of collecting and transporting used oil from a set of specific clients in the Metropolitan Region of São Paulo to a single collection centre. More specifically, the company's case is approached using concepts of the Vehicle Routing Problems (VRP) class.

1.1 Partnership and Motivations for this Thesis

This study was carried out in partnership with a Brazilian company operating in the market of used oil collection and recycling. It was responsible for providing enough information about its customers and process of collection in order to contextualize and define the problem at hand. To keep confidentiality about the company and its customers, no names or revealing details about the entities are mentioned in this study.

There are three main motivations which led to the development of this study. One of them is to understand the complex logistics of a real-life operation using the concepts developed in the class of VRP. As highlighted by Drexl (2012), the relevance of connecting theory and practice in VRP has become more evident throughout the years and modeling real cases helps to identify the theoretical gaps to be explored by academic studies. Another motivation is to cooperate with and impact the partner company, in order to bring attention to legal and ethical operations in the activity of used oil disposal, collection and recycling. It may afterwards serve as a model for other organizations which want to bring their processes closer to circular economy approaches. The third motivation is to apply concepts which were learned during the author's graduation course in a complex logistics problem.

1.2 Objectives and Contributions of this Thesis

The objectives of this graduation thesis are the following:

I To study the broad class of Vehicle Routing Problems, focusing on a specific combination of variants, and to apply the Operations Research methodology to solve a real-life problem;

II To build a mathematical model which represents the logistics operation of the fleet of trucks used by the partner company to transport the used oil from its clients to its collection centre in the Metropolitan Region of São Paulo;

III To implement the model in a computational language in order to solve the problem and find candidate solutions which could improve the current operation of the company.

As for the main contributions of this thesis, they are the following:

I Building an original model to embed Multi-Trips and Access Time Windows in the mathematical representation of the studied VRP;

II Achieving routing solutions with lower fuel consumption than the ones currently used by the partner company with different vehicle assignments.

1.3 Structure

The thesis is organized according to the following structure:

- Chapter 1 - Introduction - Presentation of the thesis' theme, its motivations, objectives, contributions and structure;

- Chapter 2 - Theoretical Background - Review on studies which served as the basis for the mathematical model, covering concepts such as reverse logistics, circular economy and VRP variants;

- Chapter 3 - Problem Description - Presentation of the routing case study as it arises in the partner company;

- Chapter 4 - Mathematical Model - Adaptation of the case to a VRP model, presenting the used notation and the model itself, along with explanatory comments to the model;
- Chapter 5 - Model Implementation - Explanation of how the model was translated into a computational language to find the best results to the problem at hand;
- Chapter 6 - Computational Tests - Tests to evaluate the model's performance both in scenarios created by a combination of factors to stress the model and in real-life scenarios, with results and discussions;
- Chapter 7 - Conclusion and Future Research - Closure of the thesis, with a summary of the work, its limitations and future research, and final considerations.

2 THEORETICAL BACKGROUND

This chapter performs a review over topics which will set the theoretical base to develop the mathematical model to the problem considered in this study. First, studies on the topics of reverse logistics and circular economy, intrinsically related to used oil collection, are discussed. Then, there is an introduction to the Operations Research methodology and to the class of Vehicle Routing Problems and its solution methods, followed by a deep review on the important variants to the problem at hand.

2.1 Reverse Logistics

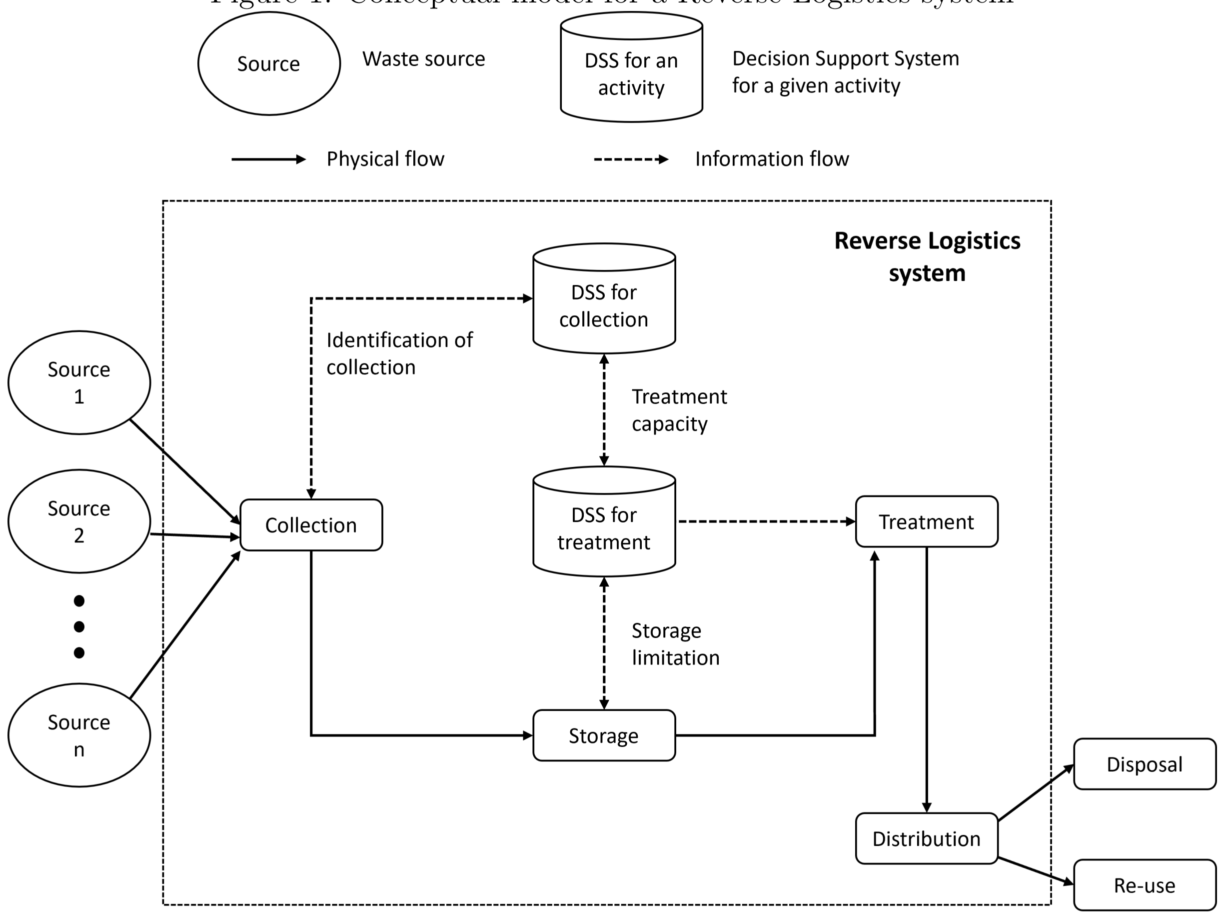
The collection of used oil for recycling can be categorized as a reverse logistics activity, which has been given slightly different definitions over the years. Some of them are listed below to illustrate the matter.

- As defined by Stock (1998), Reverse Logistics is "the role of logistics in product returns, source reduction, recycling, materials substitution, reuse of materials, waste disposal and refurbishing, repair, and remanufacturing";
- According to the Council of Logistics Management, Reverse Logistics is defined as "the term often used to refer to the role of logistics in recycling, waste disposal, and management of hazardous materials; a broader perspective includes all relating to logistics activities carried out in source reduction, recycling, substitution, reuse of materials, and disposal (BRITO; DEKKER, 2002);
- The definition of Reverse Logistics from Murphy and Poist (1988) also applies; they defined reverse logistics as "the movement of goods from a consumer towards a producer in a distribution channel".

Waste Management, a concept which encompasses the collection of used oil, is also in the category of Green Supply-Chain Management (GrSCM), defined by Srivastava (2007) in his extensive literature review on the matter as “integrating environmental thinking into supply-chain management, including product design, material sourcing and selection, manufacturing processes, delivery of the final product to the consumers as well as end-of-life management of the product after its useful life”. Most of the reviewed papers on Waste Management in that study are concerned with mathematical modelling of the waste management system, using multi-objective functions. Some examples are Caruso, Colorni and Paruccini (1993), which model a solid waste management system; Giannikos (1998), which focuses on locating facilities and the transport network; and Mourão and Amado (2005), who modelled a refuse collection case with a mixed capacitated arc routing problem approach.

Hu, Sheu and Huang (2002) proposed a model for cost minimization of a hazardous-waste reverse logistics system. The case upon which the model was tested is composed by five international high-technology manufacturers located in the Nan-Tzi Industrial Processing Zone in Taiwan, which planned to create a single organization to treat their hazardous waste. Although the case in that study is much more complex than the one in this thesis and the model considers more than just routing costs (such as treatment and storage costs), the conceptual model developed by the authors is useful to illustrate the case. Figure 1 represents that model.

Figure 1: Conceptual model for a Reverse Logistics system



Source: Hu, Sheu and Huang (2002)

In the present study, “collection” is performed by the company’s fleet of trucks, “storage” occurs in one of the 17 collection centres, “treatment” takes place in the refinery centre in Lençóis Paulista and “distribution” is carried out by the same fleet of trucks from the “collection” stage. This process is further discussed in detail in Chapter 3.

2.2 Circular Economy

Another important concept that encompasses used oil recycling is the one of “Circular Economy”. Stahel (2016) explains that there are three types of industrial economies: linear, circular and performance. The linear one turns “natural resources into base materials and products for sale through a series of value-adding steps”. The circular one is based on reprocessing goods and materials, which “generates jobs and saves energy while reducing resource consumption and waste. In a circular economy, the objective is to maximize value at each point in a product's life". Lastly, the performance economy "goes a step further by selling goods (or molecules) as services through rent, lease and share business models".

The report "Towards Circular Economy" from the Ellen MacArthur Foundation poses an interesting approach upon the subject and how to transition to a circular economy. Throughout most of its history, the industrial economy has been based on the linear model, in a "take-make-dispose" model. Although efficiency of processes has always been the focus to reduce waste, the linear model is inherently subject to many losses in the value chain. The Sustainable Europe Research Institute (SERI) estimates that, each year, the manufacturing of products in OECD countries consumes over 21 billion tonnes of materials that aren't physically incorporated into the products themselves . However, that is not the only type of waste in that model. Re-usage levels of products after the end of their first functional life are still low: in Europe, only 40% of the 2.7 billion tonnes of generated waste were reused, recycled, or composted and digested. Other losses present in the linear model are energy losses (i.e. the energy which is not recovered when disposing material) and losses related to deterioration of ecosystems (FOUNDATION, 2013).

The report presents a set of principles of the circular economy, which are summarized below:

- **Design out waste** - design the components of a product so it can fit a cycle. Biological components can be composted, while technical components (i.e. man-made materials) can be reused;
- **Build resilience through diversity** - while linear models are focused on efficiency, circular economy models are based on complex systems with many connections, which leads to lower exposure to external shocks;
- **Rely on energy from renewable sources** - although it is not essential for the success of a circular economy, the usage of more renewable sources should be put as an equally important objective;
- **Think in 'systems'** - system thinking favours the mindset of considering long term effects of every action, instead of focusing on the short term;
- **Waste is food** - biological nutrients should be reinserted on the biosphere, while technical nutrients can be re-transformed into high-quality inputs via a process called upcycling.

Some of those principles are evidently present in the base oil and used oil cycle. For example, upcycling is used to transform the used oil back again into a raw material for high-quality lubricant. The cycle is also based on a well-defined system, composed by different companies which use the lubricants, collect the used oil and recycle it back into base oil.

Although one could argue that the base oil and used oil cycle is a successful example of circular economy, moving towards a more generalized circular economy requires the involvement of several actors. Researchers and scientists need to assess benefits and impacts of products and design products to be reusable; communication channels should advertise the importance of circular economy; policymakers should encourage the creation of laws and taxes which favour circular economy; societal wealth should be measured in terms of capital instead of sales (STAHEL, 2016).

Actions at a governmental level can already be seen in some countries. The European Commission, for example, adopted the new Circular Economy Action Plan (CEAP) in March 2020, which is part of the European Green Deal. CEAP aims at achieving circularity in sectors with high potential for that, such as electronics and Information and Communications Technology (ICT), batteries and vehicles, packaging, plastics, textiles, construction and buildings, food, water and nutrients. The plan is based on a list of 35 categories of actions which will be carried out until 2023. Laws that dictate what should be done with used oils are determined in the Waste Framework Directive, which states that Member States are responsible for ensuring the treatment of used oils (COMMISSION, 2020).

2.3 Operations Research and the Class of Vehicle Routing Problems

Winston and Goldberg (2004) define Operations Research (OR) as a "scientific approach to decision making that seeks to best design and operate a system, usually under conditions requiring the allocation of scarce resources". Similarly, the International Federation of Operational Research Societies (IFORS) defines it as "the development and the application of a wide range of problem-solving methods and techniques applied in the pursuit of improved decision-making and efficiency, such as mathematical optimization, simulation, queueing theory and other stochastic models" (IFORS, 2021). The term "Operations Research" was coined during World War II when British military leaders asked scientists to study problems such as management of convoy, bombing, antisubmarine and mining operations (WINSTON; GOLDBERG, 2004). Since then, OR has evolved to cover a much broader range of applications. To cite some examples, it can be used to analyze and optimize problems of allocation of people and machines, project scheduling, production planning, budget allocation, risk management, urban planning and other situations. It is the foundation for softwares used by companies to find the best solution in a given scenario, which leads to sustainable competitive advantage.

Winston and Goldberg (2004) summarize the process of applying OR to a situation in a "seven-step model building procedure":

1. **Formulate the problem** - Specify the objective and what should be studied to solve the problem;
2. **Observe the system** - Collect data to understand what parameters affect the problem;
3. **Formulate a mathematical model** - Translate the problem into a well defined mathematical formulation;
4. **Verify the model and use it for prediction** - Determine if the model is an accurate representation of reality and, if it is, input the data from the problem;
5. **Select a suitable alternative** - Choose the alternative that best suits the objectives;
6. **Present results and conclusion** - Present the results to the decision makers;
7. **Implement and evaluate recommendations** - Implement the recommendations and monitor the system.

This methodology can be applied, for instance, in the class of VRP, which is the focus of this study.

The Vehicle Routing Problem was introduced in 1959 by Dantzig and Ramser in the paper "The Truck Dispatching Problem" as a generalization of the Travelling Salesman Problem (TSP). As expressed by Laporte (2009), the VRP can be defined as "the problem of designing least-cost delivery routes from a depot to a set of geographically scattered customers, subject to side constraints". It is known that "almost all vehicle routing and scheduling problems are NP-hard and hence unlikely to be solvable in polynomial time", as explained in Lenstra and Kan (1981). Given the demonstrated complexity of the problem and its flexibility to model and solve real cases, VRP has caught the interest of researchers and companies, leading to a deep advancement of the field since its conception.

The main idea behind VRP, just as in any optimization problem, is to get the best value for a specific metric given a set of constraints. Specifically, VRP is used to deal with the logistics planning of a company, producing, for instance, daily routes for a given fleet. In most cases, the objective is to achieve the lowest cost possible for the operation, but that is not the only metric that can be used. Other factors have been considered in different studies to cover a rich variety of problems. For example, some models may focus on time reduction, profitability, service quality, service and workload equity, consistency, simplicity, reliability and externalities (VIDAL; LAPORTE; MATL, 2020).

Not only the scope of objectives has been enlarged over the years, but also the one of constraints. For example, some classic adaptations of the original problem use time windows, pickup and delivery, heterogeneous fleet, site dependency and multiple depots. What makes the VRP a rich class of problems to be studied is that it can be modified with different combinations of objectives and constraints. Nowadays, it plays major roles for solving real logistics problems for companies such as Coca-Cola and AB Inbev (BRAEKERS; RAMAEKERS; NIEUWENHUYSE, 2016).

Most academic papers on VRP are interested in developing a new algorithm to either solve known instances which were explored by other authors or to solve a completely new problem. Over the years, authors have implemented solutions based on exact approaches, heuristics, metaheuristics and matheuristics. The last three, however, have received more attention, given that they are more suited to solve NP-hard problems such as the ones in the VRP class.

The books from Toth and Vigo (2014) and Golden, Raghavan e Wasil (2008) perform a thorough review on the VRP class, its variants and solution methods. More specifically, literature reviews on exact algorithms for VRP can be found in Christofides, Mingozzi and Toth (1981), Laporte and Nobert (1987) and Baldacci, Toth and Vigo (2010). Given that the purpose of this work is to solve the problem using a commercial solver, which uses an exact approach based on state-of-the-art Branch-and-Cut algorithms, it is not necessary to further describe different solving methods. However, for heuristics and metaheuristics methods, one could refer to Golden, Magnanti and Nguyen (1977), Laporte and Semet (2002), Bräysy et al. (2008) and Vidal et al. (2013), which are only a few examples of literature review studies. For matheuristics algorithms, one could refer to Doerner and Schmid (2010) and to Archetti and Speranza (2014).

2.3.1 Introduction to the VRP Class

The book "Vehicle Routing: Problems, Methods and Applications", by Toth and Vigo (2014), is a well-known reference for VRP studies. It defines the basic problem and its variants and surveys different solution methods for each case. It starts by giving a thorough definition of the Capacitated Vehicle Routing Problem (CVRP), which serves as a base for the mathematical model of the CVRP which will be set in this section. The CVRP notation and model present in this section are from that book. It also serves as the basis for more complex models.

In the CVRP, there is a single depot, from which goods are distributed to n customers. The depot is defined as node 0 and customers belong to the node set $N = \{1, 2, ..., n\}$. Each customer $i \in N$ has a predefined demand q_i. The distribution (or collection) is performed by a fleet of $|K|$ homogeneous vehicles, all having the same capacity Q > 0. When a vehicle moves from node i to node j, there is an incurred cost of c_{ij}.

An undirected graph can be used to represent the problem when the cost c_{ij} is equal to c_{ji}. The nodes are defined by the set $V = \{0\} \cup N$, that is, the depot plus all customers. The edges in the undirected graph can be defined as the set $E = \{e = \{i, j\} = \{j, i\} : i, j \in V, i \neq j\}$. The graph $G = (V, E)$ is formed by the nodes and edges. Given the graph, the costs c_{ij} and demand q_i, the CVRP is defined by the weighted graph $G = (V, E, c_{ij}, q_i)$ and the fleet K with capacity Q for each vehicle. For the cases where $c_{i,j} \neq c_{j,i}$, a digraph $G = (V, A)$ is used, where $A = \{(i, j) \in V \times V : i \neq j\}$.

A route is defined by a sequence $r = (i_0, i_1, i_2, ..., i_s, i_{s+1})$ in which a set of customers $S = \{i_1, i_2, ..., i_s\} \subseteq N$ is visited by a single vehicle. Note that $i_0 = i_{s+1}$ represent the depot. The route r has cost $c(r) = \sum_{p=0}^{s} c_{i_p,i_{p+1}}$. Feasibility is defined by capacity constraint $q(S) := \sum_{i \in S} q_i \leq Q$ and by the fact that each customer is visited exactly once. In this case, $S \subseteq N$ is a feasible cluster. A solution is formed by $|K|$ feasible routes, one for each vehicle $k \in K$. The routes $r_1, r_2, ..., r_{|K|}$ and the corresponding clusters $S_1, S_2, ..., S_{|K|}$ provide a feasible solution for the CVRP if all routes are feasible and the clusters form a partition of N. It is worth noting that it is possible to find a solution in which some trucks are not used. In that case, the routes for those trucks are formed by the sequence (i_0, i_{s+1})

For the case of undirected graphs, given $S \subseteq V$ a subset of vertices, the *cut set* $\delta(S) = \{\{i, j\} \in E : i \in S, j \notin S\}$ is the set of edges with exactly one or both endpoint(s) in S. For directed graphs, $\delta^-(S) = \{\{i, j\} \in A : i \notin S, j \in S\}$ are the *in-arcs* and $\delta^+(S) = \{\{i, j\} \in A : i \in S, j \notin S\}$ are the *out-arcs*. For any customer subset S, $r(S)$ is the minimum number of vehicle routes needed to serve S.

Once the notation has been introduced, it is possible to approach one of the many possible formulations of the CVRP using a directed graph. The one showed here is the *two-index vehicle flow formulation.* In that formulation, the decision variables $x_{i,j}$ assume value 1 if the vehicle goes from node i to node j, and 0 otherwise (TOTH; VIGO, 2014).

Objective Function:

$$Minimize \sum_{(i,j)\in A} c_{i,j}x_{i,j} \tag{2.1}$$

Subject to constraints:

$$\sum_{j\in\delta^+(i)} x_{i,j} = 1, \forall i \in N \tag{2.2}$$

$$\sum_{i\in\delta^+(j)} x_{i,j} = 1, \forall j \in N \tag{2.3}$$

$$\sum_{j\in\delta^+(0)} x_{0,j} = |K|, \forall j \in N \tag{2.4}$$

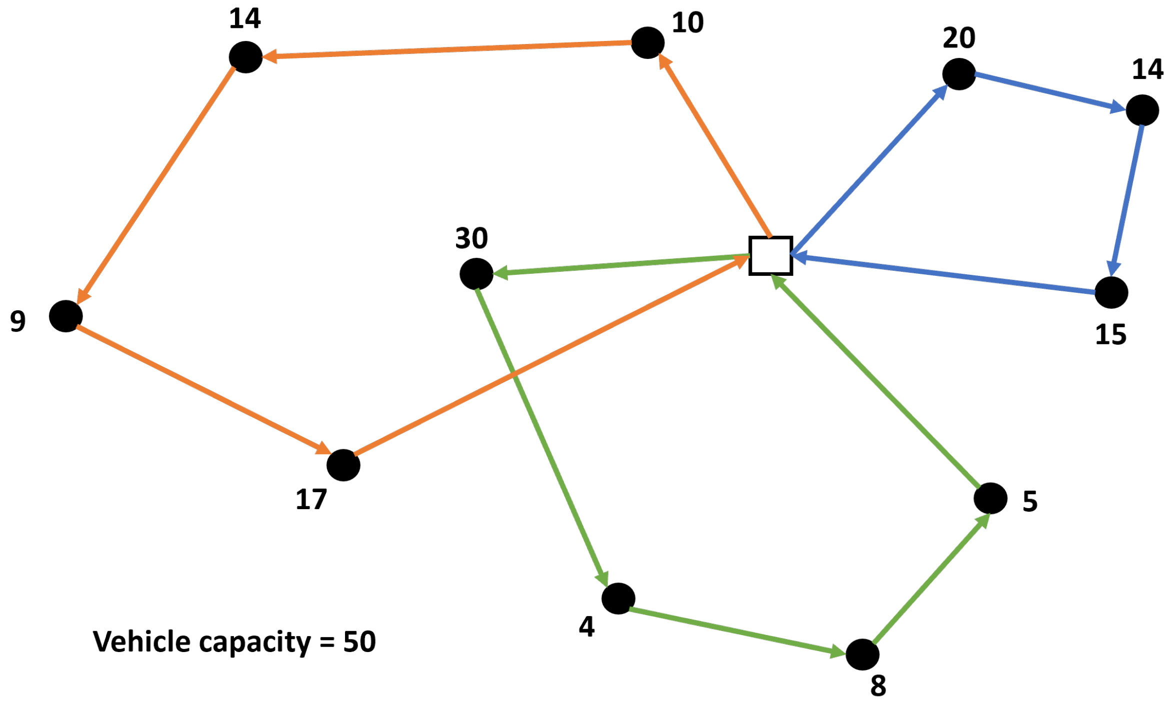
$$\sum_{(i,j)\in\delta^+(S)} x_{i,j} \geq r(S), \forall S \subseteq N, S \neq \varnothing \tag{2.5}$$

$$x_{i,j} \in \{0,1\}, \forall (i,j) \in A \tag{2.6}$$

Objective function (2.1) minimizes the routing costs. Constraints (2.2) and (2.3) state that each customer is connected to exactly two other vertices in a route. Constraints (2.4) ensure that $|K|$ routes are created. Constraints (2.5) serve both as capacity constraints and subtour elimination constraints. Constraints (2.6) define the domain of the decision variables $x_{i,j}$.

Figure 2 shows a simple but useful representation of a CVRP solution using three vehicles with few customers. The black points represent the customers, while the white square is the depot. Each customer has a demand of units to be fulfilled, represented by the number by the side of the point. The coloured edges represent the routes in the given solution, one colour for each vehicle. All vehicles have a capacity of 50 units.

Figure 2: Representation of a solution for the CVRP



Source: The author (2021)

As it is possible to see, all routes begin and end at the depot. No vehicle exceeds its capacity of 50: the orange vehicle delivers 50 units, the blue vehicle delivers 49 units and the green vehicle delivers 47 units.

2.3.2 VRP in Real-life Applications

Scientific research on VRP can be divided in two main categories: studies focusing mainly on theoretical models and solution algorithms; and Rich Vehicle Routing Problems (RVRP), a recent nomenclature used for applying VRP in real-life cases. The "rich" aspect comes from the fact that the adaptation to a real case normally requires the combination of several different types of VRP (LAHYANI; KHEMAKHEM; SEMET, 2015).

Regarding the first category of studies, it is possible to present the following variants of VRP: with time windows (VRPTW), with split-deliveries (VRPSD), pickup-and-delivery (PDP), periodic (PVRP), heterogeneous fleet (HVRP), with site-dependency (SDVRP), with Multi-Trip (MTVRP), stochastic VRP (SVRP) and dynamic VRP (DVRP). These are only some examples of problems, which have been explored by several authors. It is not an exhaustive list of topics, but it is enough to demonstrate how flexible VRP modelling can be, and it is a signal of the importance of these problems for the academic community. Most of the studies nowadays which are inside that category focus on developing new algorithms, testing them with a commonly used set of instances and comparing them to previous benchmarks in order to achieve best-known results in shorter computational time. Other studies also focus in achieving new best-known solutions. These can be variants of exact, heuristics, metaheuristics, and other types of algorithms (DREXL, 2012).

This thesis falls in the second category, of Rich VRP, given that its focus is not to develop a new algorithm for a known problem, but to combine different variants of VRP to solve a real case study. Lahyani, Khemakhem and Semet (2015) provide a taxonomy on RVRP based on some papers that approached that topic since 2006. The authors surveyed a total of 41 papers in a non-exhaustive classification using relevant features to distinguish the papers. Those features are divided in two groups: Scenario Characteristics (SC) and Problem Physical Characteristics (PPC). In the first group, the attributes used to classify papers were *Input data*, *Decision management components*, *Number of depots*, *Operation type*, *Load splitting constraints*, *Planning period*, and *Multiple use of vehicles*. In the second group, the attributes were *Vehicles* (Type, Number, Structure, Capacity constraints, Loading policy and Drivers regulations), *Time constraints*, *Time windows structure*, *Incompatibility constraints*, *Specific constraints*, and *Objective function*.

Some of their findings show that among the surveyed papers, 70.7% are focused on pickup and delivery problems, more than half take time windows into consideration and 53.7% present a specific restriction related to the real-life application. This is an indication of how a RVRP tries to approximate real-life cases by extending the original CVRP. After classifying all 41 papers, the authors proceed to divide them into clusters according to assigned scores for the amount of attributes present in the SC and PPC groups. The proposed definition of RVRP by the authors is the following: “A RVRP extends the academic variants of the VRP in the different decision levels by considering at least four strategic and tactical aspects in the distribution system and including at least six different daily restrictions related to the physical characteristics. When a VRP is mainly defined through strategic and tactical aspects, at least five of them are present in a RVRP. When a VRP is mainly defined through physical characteristics, at least nine of them are present in a RVRP” (LAHYANI; KHEMAKHEM; SEMET, 2015).

Usually, in order to apply VRP concepts to solve logistics challenges, companies buy a software that provides that service without requiring great programming or academic knowledge from the operator. Drexl (2012) compares research with real applications by studying Commercial Vehicle Routing Softwares (CVRSs) in some companies. The author defines a CVRS as “a computer program that allows to (1) read in and display data on vehicle depots, customers, distances, and travel times between locations, on requests, vehicles, and drivers, (2) construct, save, and display vehicle routes, and (3) determine a complete route plan for a given data set (a problem instance) by executing construction and improvement algorithms, possibly after entering a set of parameters, without further user interaction” (DREXL, 2012). The author mentions some reasons that justify the importance of a CVRSs, such as reduction of planning costs and of repetitive tasks, improvement in routing quality and surveillance and less dependence on a single person that has the knowledge for routing.

Drexl (2012) sent a questionnaire to companies in the German market which use CVRS and got 28 responses. Most of them considered the algorithm as a core competence and more than 75% stated that they cooperated with at least one university to develop the software, which highlights the importance of connection between theory and practice. After a research with the firms that produce the CVRSs, the author also enumerated some of the usages of software, such as the sourcing of raw materials and transport of semi- and finished goods to warehouses and wholesalers, distribution of goods to the final customer, routes for freight forwarders, letter mail services, reverse logistics, waste collection, staff dispatching problems and intra-plant logistics. These are only some examples of the applications and importance of CVRS in real practice logistics, which evidences how flexible the adaptation of VRP to companies can be.

2.4 VRP Variants

The rest of this chapter is dedicated to the review on some of the most important VRP variants, which were used as a basis to develop the model which describes the problem studied in this thesis. The studied variants are the following: VRP with Time Windows, with Heterogeneous Fleet, with Site Dependencies, with Multi-Trips and with Access Time Windows. At the end of the chapter, a subsection with the summary of all reviewed studies is presented.

2.4.1 VRP with Time Windows

VRP with time windows (VRPTW) is one of the most classic and well-known variants of the CVRP. It adds the condition that customers have a restricted time period in which they can be served. The time windows can be hard (when no violation of the time window can occur) or soft (when the violation is allowed, but comes with a penalty). The first type is the most used and studied, because it is a good approximation for real scenarios (for example, deliveries for banks, grocery stores, postal services and others).

The wideness of the time windows influences a lot the feasibility of the solutions. For example, when time windows are narrow relative to the planning horizon, there is a more restricted amount of solutions which are feasible; when they are very wide, the problem may turn almost into an ordinary CVRP, given that the time windows may not be a rigid constraint. This also influences the solution method, which can be exact or based on heuristics and metaheuristics (DESAULNIERS; MADSEN; ROPKE, 2014).

Desaulniers, Madsen and Ropke (2014) do a thorough analysis of VRPTW, reviewing different mathematical formulations and exact and heuristic solutions. For the exact solutions, the focus is on methods developed after the year 2000, such as Branch-and-Cut, Branch-and-Cut-and-Price, and reduced set partitioning. For a revision of methods developed before 2000, one could read the review of Cordeau et al. (2002).

The study of VRPTW gained more attention after Solomon (1987) created benchmark instances with up to 100 customers. The author tested different heuristic algorithms on the instances, such as based on savings, nearest neighbors and insertion. From then on, these instances started to be used by other authors researching VRPTW to test their algorithms. One of the most used exact algorithms for the VRPTW is Branch-and-Price, which uses the methodology of column generation. It was first used in a VRPTW by Desrochers, Desrosiers and Solomon (1992). The Branch-and-Cut-and-Price adds cutting planes to strengthen the LP relaxation of the problem. Branch-and-Cut works with a similar approach of using planes to tighten linear relaxation, but it doesn't use the column generation approach. Reduced set partitioning solutions use a particular formulation of the VRPTW and aim to preset a large number of variables to be equal to 0 and then solve the reduced problem using a commercial solver, such as CPLEX (DESAULNIERS; MADSEN; ROPKE, 2014). Baldacci, Toth and Vigo (2010) and Baldacci, Mingozzi and Roberti (2011) successfully applied this method.

Desaulniers, Madsen and Ropke (2014) compare three exact approaches using three papers which apply the methods over Solomon's instances. Jepsen et al. (2008) and Desaulniers, Lessard and Hadjar (2006) use the Branch-and-Cut-and-Price approach, although the latter also applies Tabu Search in the column generation step to accelerate that process. Baldacci, Mingozzi and Roberti (2011) used the reduced set partitioning methodology. Comparing all three, the last one presents the best results, both in computational time and number of instances solved.

Regarding heuristics, most of VRPTW use variants of local search methods. One of the most crucial points when developing a heuristic algorithm is to define the neighborhood structure. When taking that decision, one could choose between a *traditional* neighborhood, "whose size is growing polynomially with n in a controlled manner such that all solutions in the neighborhood can be evaluated by explicit enumeration" (DESAULNIERS; MADSEN; ROPKE, 2014), n being the number of customers, or a *large* neighborhood, which cannot be searched explicitly due their size. In the first one, all solutions in the neighborhood are evaluated, while in the second one, normally a heuristic method is applied to find good candidates, such as the *Large Neighborhood Search* (LNS) framework, popularized by Bent and Hentenryck (2004). In that paper, the authors use a 2-step approach, first applying Simulated Annealing to get the lowest amount of vehicles for the problem and then the LNS to minimize the total travelled distance. LNS's principle is to *destroy* and *repair* solutions in iterations until a stopping criterion is met. In this context, to destroy a solution means to remove a subset of customers, and to repair it means to reinsert those customers in a different position of the route (DESAULNIERS; MADSEN; ROPKE, 2014).

Metaheuristics can also be based on population search, another family of algorithms. The main idea is to keep and update a pool of solutions, finding better solutions by altering that pool. One example of population-based search is the class of Memetic Algorithms. The idea is to repeatedly combine, mutate and apply local search over the solutions of the pool, creating a new pool. Nagata, Bräysy and Dullaert (2010) apply a Mememtic Algorithm over the VRPTW.

2.4.2 VRP with Heterogeneous Fleet

This is another well-known variant of the CVRP. When a VRP involves different types of vehicles, it is called a Mixed Fleet or a Heterogeneous Fleet VRP (HVRP). The impact over the model is mainly due to those different types having different costs and capacities. The HVRP can be used in different scenarios. Koç et al. (2016) perform a summary of the main types of HVRP, using the following classification:

1. Fleet Size and Mix Vehicle Routing Problem (FSM);
2. Heterogeneous Fixed Fleet Vehicle Routing Problem (HF).

The FSM first appeared in Golden et al. (1984), with the objective of answering the question "How many and what size vehicles are needed in order to accommodate demand at minimal cost?". The problem consists in determining the size of a certain fleet, considering both variable costs of operating a vehicle and fixed costs of leasing those vehicles. Additionally, it allows for a heterogeneous fleet to be formed, instead of one where all the vehicles have the same characteristics.

The mathematical formulation is composed by an objective function which aims at minimizing the sum of fixed costs of activating a vehicle and variable costs, proportional to the distance traveled, and constraints, which are similar to those of the CVRP. The difference is that an infinite supply of vehicles is assumed, which allows for the best sizing of the fleet to occur. The authors focus on developing and testing different algorithms to solve the FSM.

Taillard (1999) introduced the HF variation. It deals with a problem which is more similar to the one in this thesis, which is solving a VRP instance with a given fixed heterogeneous fleet, instead of sizing it. The authors use a heuristic column generation method to solve the HF. The mathematical model considers only variable costs for the trucks, and the solution is based on solving a homogeneous VRP for each type of vehicle and then combining the solutions to solve the whole problem. Taillard (1999) also extends the formulation to approach the FSM and compares the results with the ones in Golden et al. (1984) and other studies which claimed to be the best solutions at the time the study was published. It shows a clear improvement provided by the algorithm used by the authors.

Regarding the HF, most solutions are achieved via heuristics and metaheuristics, given that it is a harder problem to solve than the FSM. Current papers focus on finding better algorithms to find faster solutions to the problem. Tarantilis, Kiranoudis and Vassiliadis (2004) were inspired by Taillard to developed an algorithm called BATA to solve the HF problem. BATA is an acronym for backtracking adaptive threshold accepting method. It is a local search-based metaheuristic that belongs to the class of threshold accepting algorithms. As previously explained, the idea of this type of algorithm is to allow "moves towards solutions with higher objective function values (uphill moves) in order to escape from local minima" (TARANTILIS; KIRANOUDIS; VASSILIADIS, 2004). The difference in BATA is that the threshold is not necessarily reduced monotonically. It could perform small increases during optimization, which, according to the authors, results in an oscillating strategy that achieves a dynamic balance between diversification and intensification of the search process. They used instances and results from Taillard (1999) as benchmark to compare the performance of the algorithms. BATA's results show an average improvement of 0.31% with respect to the algorithm from Taillard (1999).

Li, Golden and Wasil (2007) adapted a Record-to-Record algorithm used in previous works which solved the HF problem. The new algorithm was named HRTR. It also follows the idea of allowing uphill moves in order to escape from local minima. HRTR's results proved to be competitive when compared to BATA's, showing even new best solutions for some of the instances.

Euchi and Chabchoub (2010) bring two contributions to HF solving: a new Tabu Search algorithm combined with an Adaptive Memory Procedure (AMP), and a set of three initial solutions, which are consecutively improved by the algorithm. Basically, the AMP keeps visited solutions stored in memory, and new solutions are created by improving the ones in the memory, resulting in an iterative process. Euchi and Chabchoub (2010) results are compared to the ones in Taillard (1999), Tarantilis, Kiranoudis and Vassiliadis (2004) and Li, Golden and Wasil (2007), which are commonly used for benchmarking HF solutions, and they show substantial improvement both in computational time and closeness to the best known solution.

Brandão (2011) also approaches the problem with a Tabu Search algorithm, giving a detailed description of the procedure. When compared to benchmarks, the results in Brandão (2011) were better than in Tarantilis, Kiranoudis and Vassiliadis (2004) and Li, Golden and Wasil (2007), but worse than in Euchi and Chabchoub (2010).

2.4.3 VRP with Site Dependencies

In the Site-Dependent VRP (SDVRP), the original CVRP is altered by considering that each customer can only be served by a subset of vehicles. It is normally combined with the HFVRP, in which case the subsets of vehicles are formed by vehicles of different types. The SDVRP approaches real-life conditions primarily for urban logistics in which, for example, some customers are not able to receive a specific type of truck due to location or space constraints.

One of the pioneers in studying the SDVRP is Nag (1988), in which four methods are developed to solve the problem, named VEHTYPE, GAP1, GAP2 and GAP3. VEHTYPE uses sweep and savings concepts and solves to one vehicle at a time, from the smallest to the largest type. GAP1 and GAP2 use VEHTYPE as seed solutions and improve the results by applying a generalized assignment approach, although GAP2 uses a simultaneous approach of assignment instead of sequential. GAP3, on the other hand, matches the largest vehicles to the customers which have compatibility with them and it works its way down to the smallest type. The authors test the algorithms with 6 groups of customers, combining different quantities (50, 75 and 100 customers), amount of vehicles and different customer compatibility. All algorithms performed under 10 seconds and, according to the authors, the best algorithms overall were GAP2 and GAP3, although there were no clear “winners”.

Chao, Golden and Wasil (1999) developed a better heuristic algorithm after studying the ones in Nag (1988), starting from a feasible solution and improving it with a sequence of uphill and downhill moves, calling it Site-Dependent Heuristic (SDH). They created 12 different tests, which also have different amounts of customers and vehicles. They also adapted five well-known VRP tests into SDVRP and used the six from Nag (1988), totaling 23 tests. SDH improved the best-known solutions by an average of 8.4% in the existing tests, and produced good results for the other tests.

Cordeau and Laporte (2001) took a different approach, modelling the SDVRP as a Periodic VRP (PVRP). Their algorithm is also based on heuristics, starting from a random initial solution an improving it. Their results achieved either equal or better costs than in Chao, Golden and Wasil (1999).

Later, Chao and Liou (2005) developed a Tabu Search algorithm to deal with the SDVRP. In the construction phase, an initial solution is created, which is then improved using Tabu Search. The authors used the same 23 tests as in Chao, Golden and Wasil (1999), and showed better results, competitive with the results shown in Cordeau and Laporte (2001).

Alonso, Alvarez and Beasley (2008) combine the SDVRP with other adaptations of VRP: Multi-Trip VRP (MTVRP) and Periodic VRP (PVRP). They call the problem Site-Dependent Multi-Trip Periodic Vehicle Routing Problem (SDMTPVRP). The mathematical formulation of the problem and the algorithm used to solve it, a Tabu Search named TS-ABB by the authors, were inspired in Cordeau, Gendreau and Laporte (1997), which originally solved the PVRP. However, the authors also tested TS-ABB on the same test instances as in Cordeau and Laporte (2001) and Chao, Golden and Wasil (1999), showing not only better results, but also faster relative computational times.

2.4.4 VRP with Multi-Trips

In a traditional VRP instance, vehicles are allowed to do only one journey, that is, to go out and back to the depot only once. Fleischmann (1990) introduced the concept of multiple trips in VRP in order to get closer to urban real-life cases, where "demands are high and travel times short". He considered a single-depot VRP with heterogeneous fleet, time windows constraint for customers and the possibility for vehicles to go out and back to the depot more than once.

The problem regarding multiple trips is called by Fleischmann as Vehicle Routing Problem with Multiple Use of Vehicles (VRPMU). The method used to solve it was named Savings Procedure for Multiple Use of Vehicles (SPMU), a one-phase parallel Savings Procedure (SP). The author proceeded to test the algorithm in three real cases. One for a carrier delivering miscellaneous consumer goods in West-Berlin, in 1985, and the other two from the time when the paper was written: one regarding delivery of food from a depot in Duisburg/Rhein to supermarkets and the other for the delivery of beverages from a depot near Dortmund to retailers. In the last two cases, the author also considered time windows, besides multiple trips. Compared to a multi-phased or sequential traditional SP, Fleischmann's SPMU yielded better and faster results (FLEISCHMANN, 1990).

Taillard, Laporte and Gendreau (1996) also studied the Multi-Trip Vehicle Routing Problem (MTVRP). The used algorithm was based on Tabu Search, first creating a feasible set of routes and then selecting them using an enumerative algorithm. Then, those routes are assembled into working days using concepts of a bin packing algorithm.

Brandao and Mercer (1997) dealed with the MTVRP with a Tabu Search algorithm (TSAMTVRSP). It was used to deal with the real-life case of vehicle routing for a distribution centre of a British company, Burton's Biscuits Ltd. The case also involved heterogeneous fleet, with 11 vans and 11 tractors which had double the capacity of the vans. The TSAMTVRSP results were compared with the ones from an expert worker which designed the schedules. It outperformed the manual result in every measure taken, with the results being 20% better on average.

Later, Brandao and Mercer (1998) simplified the algorithm to compare it with the one by Taillard, Laporte and Gendreau (1996). Their new algorithm, called TSMTVRP, presented faster computational results, even though it is adapted to include more constraints, such as time windows and heterogeneous fleet.

Since it was first introduced in 1990 by Fleischmann, the MTVRP has received a lot of attention due to its similarity to real-life cases. A thorough survey on the topic was performed by Cattaruzza, Absi and Feillet (2016), summarizing different mathematical formulations and heuristic approaches. They also go on to survey different variants and applications of the MTVRP.

2.4.5 VRP with Access Time Windows

Among all of the discussed VRP variants in this thesis, the VRP with Access Time Windows (VRPATW) is the less considered by the literature. What is intended with "Access Time Windows" is the adaptation to a mathematical model of the real-life constraint of certain areas of a city which have a time restriction for vehicles. For example, some cities restrict their centers for vehicle circulation during a certain period of the day.

The work of Grosso et al. (2018) studies the relationship between access time windows in European cities and their impact on greenhouse gases emissions and energy consumption. The described problem in their article was the closest to one studied in this thesis in terms of Access Time Windows, which is why the model they developed served as inspiration for the one which will be shown in Chapter 4. The authors divide the city between a central restricted zone (RZ), which is under the influence of an access time window (ATW), and a non-restricted zone. The ATW during which vehicles are allowed to travel inside the RZ has an opening time and a closing time. Customers are spread between the non-restricted zone and the restricted zone, while the depot is located outside the restricted zone.

The authors developed a mathematical model to represent the VRPATW and solved instances with up to 10 customers using that model to find the optimal solution. For instances with more customers, the authors solved the problem using different heuristic and metaheuristic approaches. The model's objective function aims at minimizing both route duration and vehicle usage. The authors also use subsets of dummy nodes to represent the point of start and end of the route, one for each vehicle. Therefore, if a vehicle is not to be used according to the optimal solution, it will simply go from the dummy start node to the dummy end node, without serving any customers nor spending any time. Besides using an exact approach to solve the problem, the authors also applied three heuristic methods: an adaptation of the Clarke and Wright (1964) savings algorithm, a Genetic Algorithm and a Tabu Search algorithm. All tests were performed over a customer base of a small-size carrier in Seville, Spain. The authors did not come to a conclusion over which heuristic method had the best performance.

Mancini (2016) tackled a similar problem in her work with a different mathematical model. She studied a Vehicle Routing Problem with Mixed Fleet and Limited Traffic Zones (VRPLTZ). In that problem, the fleet is composed by two types of vehicles and customers are split between a Limited Traffic Zone (LTZ) and a Free Access Zone (FAZ). One of the vehicle types can access the LTZ at any time, while the other type can enter it only during a specific time window. Routes in the model have limited duration and the model aims at minimizing all routes' total length. The author developed a Large Neighborhood Search algorithm to solve the problem.

The limitation in the model developed in Mancini (2016) is that it does not represent the LTZ as a whole - the constraint related to the LTZ access is dealt with by using starting and finishing times for the service performed in customers which are inside the LTZ. Therefore, the problem does not cover the possibility of a vehicle violating the LTZ time window when it is on its way to attend a client inside the LTZ; it only guarantees that the service will not start during that time window.

2.4.6 Summary of the Studied VRP Variants

Table 2 presents a summary of the studies reviewed in Section 2.4. The table also presents columns with different VRP variants, in which an "X" is placed if the study considers the indicated variant. There is also a "Literature Review" column, which follows the same logic, and a "Solution Algorithm" column that highlights the used algorithm in the study. The last row of the table places comparatively the approach developed in this thesis.

Table 2: Summary of the studied VRP variants

Study	**VRPTW**	**HFVRP**	**SDVRP**	**MTVRP**	**VRPATW**	**Literature Review**	**Solution Approach**
Baldacci, Mingozzi and Roberti (2011)	X						Reduced Set Partitioning
Baldacci, Toth and Vigo (2010)	X						Reduced Set Partitioning
Bent and Hentenryck (2004)	X						LNS
Cordeau et al. (2002)	X					X	-
Desaulniers, Lessard and Hadjar (2006)	X						Branch-and-Cut-and-Price
Desaulniers, Madsen and Ropke (2014)	X					X	-
Desrochers, Desrosiers and Solomon (1992)	X						Branch-and-Price
Jepsen et al. (2008)	X						Branch-and-Cut-and-Price
Nagata, Bräysy and Dullaert (2010)	X						Memetic Algorithm
Solomon (1987)	X						Various, based on heuristics
Brandão (2011)		X					Tabu Search
Euchi and Chabchoub (2010)		X					Tabu Search
Golden et al. (1984)		X					Various, based on heuristics
Koç et al. (2016)		X				X	-
Li, Golden and Wasil (2007)		X					Record-To-Record Travel
Taillard (1999)		X					Column Generation
Tarantilis, Kiranoudis and Vassiliadis (2004)		X					Threshold Accepting-Based
Alonso, Alvarez and Beasley (2008)	X	X	X				Tabu Search
Chao and Liou (2005)		X	X				Tabu Search
Chao, Golden and Wasil (1999)		X	X				Heuristics
Cordeau and Laporte (2001)		X	X				Heuristics
Nag (1988)		X	X				Various, based on heuristics
Brandão and Mercer (1997)		X		X			Tabu Search
Brandão and Mercer (1998)	X	X		X			Tabu Search
Fleischmann (1990)	X	X		X			Savings Procedure
Taillard, Laporte and Gendreau (1996)				X			Tabu Search
Cattaruzza, Absi and Feillet (2016)				X		X	-
Grosso et al. (2018)	X	X	X		X		Exact, Clarke and Wright, Genetic, Tabu Search
Mancini (2016)	X	X	X		X		Large Neighborhood Search
This thesis	**X**	**X**	**X**	**X**	**X**		**Exact Approach**

3 PROBLEM DESCRIPTION

Once the theoretical basis to understand the VRP at hand has been set, the context of the problem can be given. The company in question is responsible for collecting 200 million liters of used oil each year from more than 45,000 clients all over Brazil, that can be any type of company which discards used oil. Some examples are mechanical workshops, gas stations, car assemblers and manufacturing plants. Both in Brazil and in the Metropolitan Region of São Paulo (MRSP), it holds around 35% of the market share for the used oil collection market, which makes it the largest player in both, while the second largest holds less than 15% of the market share in both. It collects used oil from more than 3,300 cities, brings it to one of its 17 collection centres, and then brings it to the refinery centre in Lençóis Paulista to recycle it, transforming it back to high-quality and purity oil. The problem studied in this thesis regards one of the collection centres located in Osasco, São Paulo, which is responsible for collecting used oil from customers in the MRSP.

3.1 Collection Process

When discarding oil, the company which used it must be sure that the collecting company has a document attesting for Authorization for Operation from ANP, Environmental Licenses and that it is registered in the Brazilian Institute of Environment and Renewable Natural Resources (Instituto Brasileiro do Meio Ambiente e dos Recursos Naturais Renováveis, IBAMA). After giving the used oil for transport, it should also receive a Collection of Used Oil Certificate and after the collection it should also ask for the Certificate of Final Destination, which is emitted by the refinery. Irregularly storing and discarding used oil is a severe environmental crime in Brazil. The companies and people involved in that action may have to pay substantial fines, answer for any environmental crime caused by that practice, may be suspended and even be sentenced from 1 to 4 years in jail.

The cycle of oil usage and recycling can be summarized in the following steps:

1. **Legal collection** - The collecting company uses its vehicle fleet to collect used oil from customers;

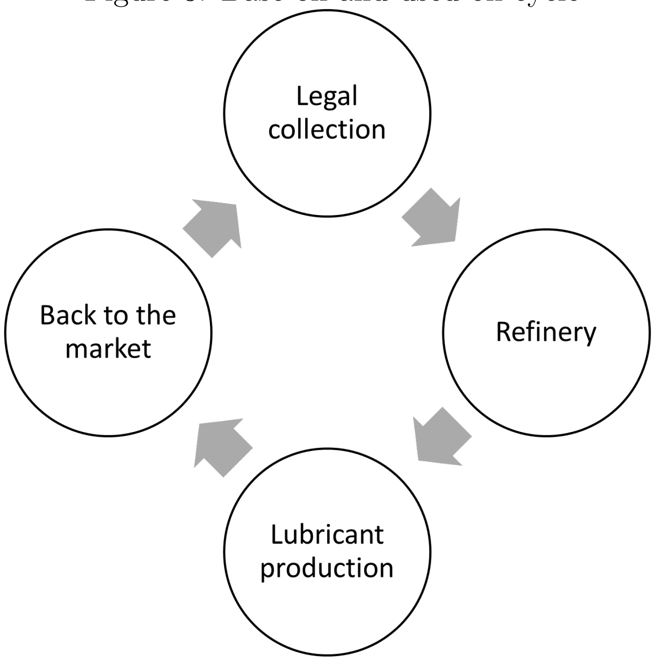
2. **Refinery** - The used oil is recycled back to base oil, used as raw material for lubricants;

3. **Lubricant production** - The base oil goes to lubricant producers, who mix it with additives to turn it back into a lubricant;

4. **Back to the market** - The lubricant returns to the market in the form of industrial, agricultural, automotive and electrical products.

This process is repeated over and over, minimizing the production of oil and encouraging sustainability and circular economy practices. Figure 3 illustrates the cycle.

Figure 3: Base oil and used oil cycle



Source: The author

3.2 Case Study

As previously explained, this thesis was done in partnership with a Brazilian company which collects and recycles used oil. More specifically, it is interested in the collection centre in Osasco, which is responsible for collecting oil from all customers inside that area. Figure 4 represents the MRSP highlighted in red.

Figure 4: Metropolitan Region of São Paulo

Source: Wikipedia page of MRSP (2021)

The problem at hand deals with key clients of the company in the MRSP, which represent the highest revenue stream for the company. For this specific group, the daily demand for collection varies from 20 to 30 customers per day approximately. The centre keeps a database of all its clients and has two ways of establishing the demand for a certain day for a certain client: either the client orders a specific amount of collection or the collecting company performs a forecast for that demand based on past collections. The required amount of time to do a collection is directly proportional to the demand volume.

The vehicle fleet responsible for this group of clients is composed by five trucks of two different types: three “small” trucks and two “large” trucks. The main difference between them is the used oil capacity that the vehicle can carry, which results in different fuel consumption rates and vehicle sizes. The “small” truck is able to carry up to 2500 liters of used oil, while the “large” one is able to carry up to 5000 liters of used oil. Small and large trucks have a fuel consumption rate of 0.14 L/km and 0.2 L/km, respectively. Regarding speed, according to the company, it can be considered that both vehicle types travel with an average speed of 35 km/h in the MRSP, using diesel as fuel.

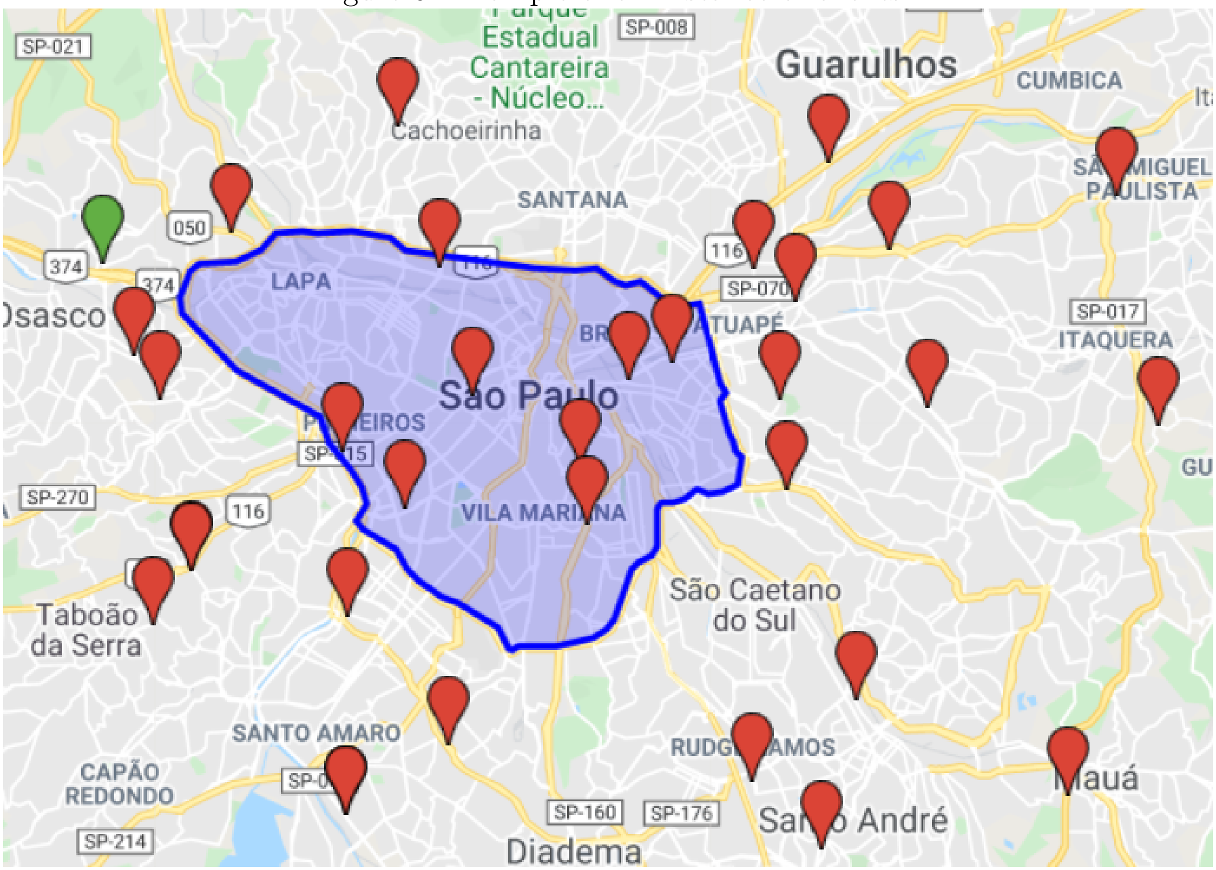
The time it takes for a vehicle to attend a client is divided in two parts: a fixed amount and a variable amount of time. The fixed part takes about 15 minutes and is related to docking the vehicle, processing documents and discussing the collection with employees from the client company. The variable amount is related to getting the used oil into the vehicle. Both types of truck use the same oil pump for collecting oil, which works at a suction rate of 6000 liters/hour. Unlike the loading time, the unloading time in the depot is considered to be a fixed amount. Although in reality there is a variable time for the unloading process, which depends on the amount of used oil the truck is carrying when it arrives at the depot, it is considered that it has little influence over the problem because the depot's pump rate is 45000 liters/h, a much higher rate than the vehicles' pump rate. Therefore, the unloading time is set to 20 minutes each time the vehicle passes through the depot, which is the time it takes for the vehicle to dock and for employees to perform sampling and analysis over the used oil that arrives at the facility.

Clients open and close at 8:00 and 16:00, respectively. The working hours of the fleet and the collection centre are from 7:00 to 19:00 in week days. A truck can be used, during a day, for a maximum of eight consecutive hours. Clients are spread all over the MRSP. Due to spatial or regional constraints, some clients cannot receive the large truck, while others are accessible by both types. Another particularity of the MRSP is that part of it works under a *traffic restriction program*, defined by the Traffic Engineering Department (Companhia de Engenharia de Tráfeco, CET). This program sets some rules for traffic in a portion of São Paulo city. This portion will be called Traffic Restricted Region (TRR) in this thesis, illustrated in Figure 5. Considering that the trucks carry a product classified as "hazardous", they are prohibited from travelling inside the TRR from Monday to Friday between 5h and 10h and between 16h and 21h (CET, 2021). Therefore, considering the functioning time of the company during weekdays, trucks can only travel inside the TRR or through its borders from 10h to 16h. Another restriction in the region is applied only for the large trucks: due to their size, they are prohibited from ever entering the TRR, although they can travel over the streets that define its borders. Figure 6 illustrates both the TRR and an example of one day of demand from the group of key clients. The red markers pinpoint the clients' location, while the green marker pinpoints the Osasco depot's location.

Figure 5: Traffic Restricted Region

Source: CET (2021)

Figure 6: Example of an instance of clients



Source: The author

Currently, the company uses a commercial software named SimpliRoute to deal with the routing. The software is a paid platform which uses input data from the company to optimize collection routes, schedule trucks and drivers, monitor the routes in real-time and backup documents from the collection (SIMPLIROUTE, 2021). Everyday, the logistics and commercial teams of the partner company gather at the end of the day to discuss the following day's collections, and the routes and vehicles needed to perform them, using SimpliRoute. According to the partner company, that activity takes an average of 1 hour to be completed. SimpliRoute's optimization does not have the objective of minimizing total travelled distance; instead, there are optimization options such as load balancing among the vehicles, minimization of the amount of used vehicles, doing only one trip per vehicle and dividing the vehicles by collection areas.

3.3 Problem Definition

Given the mentioned characteristics of the case, it is clear that the adaptation of the partner company's logistics operation to a VRP would result in a complex and interesting case. The objective, as expressed by the company, is to reduce the variable costs associated with the total travelled distance by the fleet which serves the clients with highest revenue in the MRSP, while being compliant with all constraints. Given the difference in fuel consumption between the two types of truck that compose the fleet, that variable cost to be minimized is represented by the total amount of diesel fuel consumed by the trucks.

The problem is characterized by a single collection centre from which the heterogeneous fleet of vehicles departs to attend n clients and comes back to at the end of the work shift. The vehicles can perform multiple trips in a single day. There are two types of vehicles, "small" and "large", which differ in two aspects: consumption of fuel per travelled kilometer and capacity to carry used oil. Each truck can only work a maximum of eight consecutive hours each day. The clients are spread all over the MRSP, some of them being inside the TRR and some of them being out of it, along with the depot, which is located in Osasco. The TRR can only be accessed by "small" trucks, but they cannot circulate there freely; instead, they are limited to a fixed access time window. "Large" trucks are allowed to circulate in the streets which define the TRR borders, such as Marginal Tietê and Marginal Pinheiros, but are also restricted by the access time windows in that situation. All clients and depot are also subject to their own time windows, which are related to their working hours and availability to receive the trucks for collection.

Given the contextualization and the literature review, the problem can be defined as a Time-Constrained Site-Dependent Vehicle Routing Problem with Heterogeneous Fixed Fleet, Access Time Windows and Multi-Trips applied in the context of used oil collection. It is worth noting that, according to the definition in Lahyani, Khemakhem and Semet (2015), the problem can be considered a Rich VRP. The next chapter proposes a mathematical model which encompasses all of those constraints and an objective function in order to represent the company's problem.

4 MATHEMATICAL MODEL

This chapter presents the notation used in the mathematical model, the model itself and its explanation.

4.1 Notation

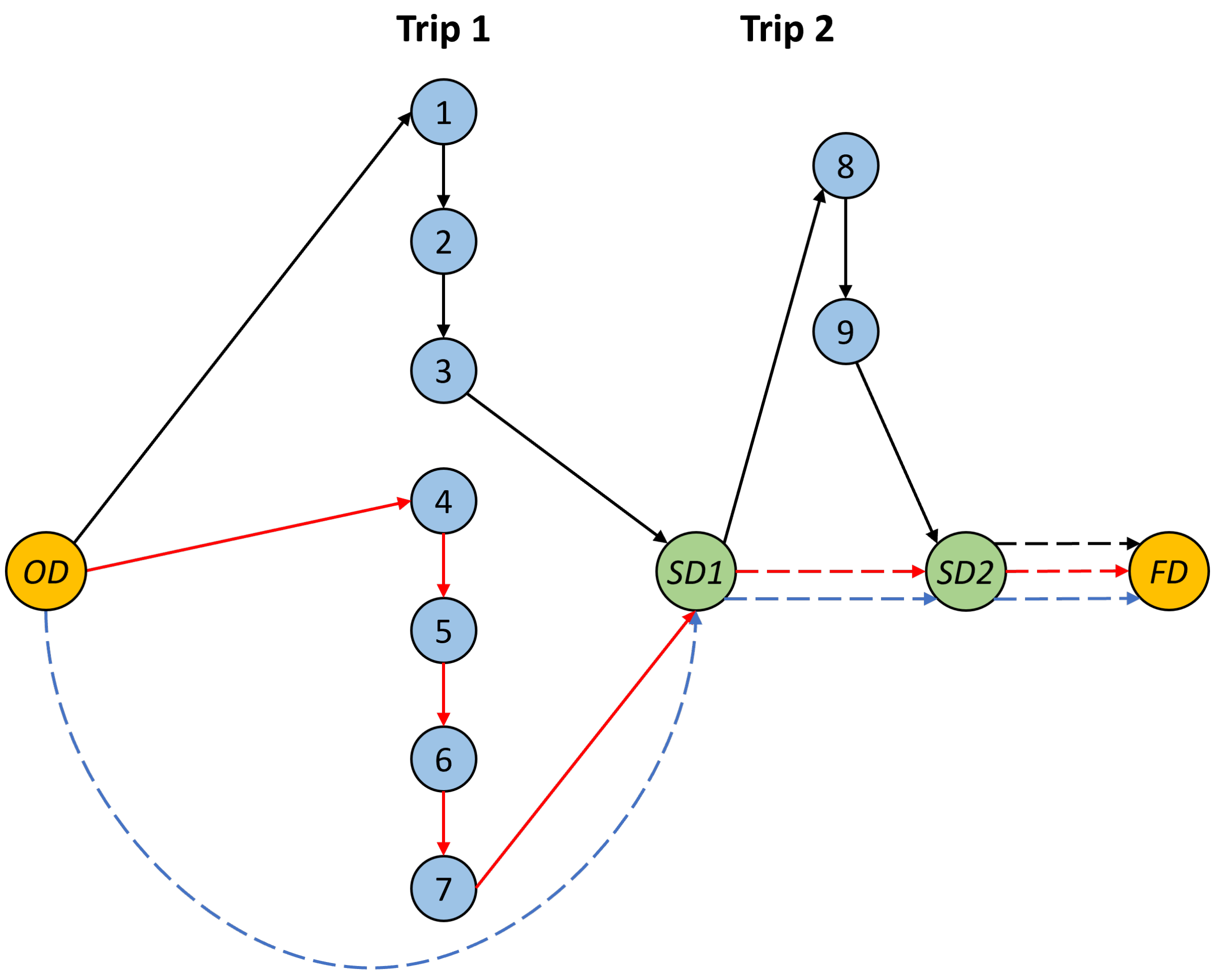
The set of clients is $C = \{1, 2, ..., n\}$. Every client i has a demand D_i of used oil that must be collected by the company. The fleet is composed by k vehicles which form the set $K = \{1, 2, ..., |K|\}$. The maximum amount of oil that a vehicle k can carry is limited by its capacity Q_k. The vehicles are also characterized by the variable cost VC_k, which is different according to the vehicle type. This variable cost is dependent on the travelled distance and represents the amount of diesel fuel consumed by the vehicle to travel a certain distance, that is, (liters of diesel)/(travelled km). Parameter T_k sets the total amount of consecutive hours a vehicle can function during a day.

The multi-trip aspect of the problem was adequately treated by using a modeling artifice, which consists in creating 3 different types of nodes to represent the collection centre, all referring to the same point in the map. The first node that represents the distribution centre is the *Origin Depot* (OD), from where all vehicles depart. Then, there are *Sink Depots* (SD), a group of nodes which has the function of absorbing the collection of the vehicles in their multiple trips. The number of sink depots is equal to the number of possible trips each vehicle can perform, thus it is defined a priori for each instance. For example, if each vehicle can perform up to three trips in a day, the number of sink depots in the model is defined as three. A *Final Depot* (FD) is also used as the final destination of every vehicle, once all demand has been collected. It is worth mentioning that since all depot nodes are located in the same point in the map, distances and travel times between them are all equal to 0. This approach to deal with multi-trips brings the problem closer to a One-to-Many-to-One variant of the Pickup-and-Delivery class of VRP, which is reviewed in Battarra, Cordeau and Iori (2014).

The depot is therefore represented by node OD, by a node set SD containing all sink depots and by node FD. Set $N = C + SD + \{OD, FD\}$ represents all nodes in the graph. N^{TRR} is a subset of N which contains only clients which are inside the TRR, while N^{NR} is a subset of N containing all nodes which are outside the TRR. There is a subset of vehicles from K named K^{LT}, formed only by large trucks. Vehicles in K^{LT} are not allowed to enter the TRR and, therefore, cannot serve clients in N^{TRR}.

Figure 7 uses a simple example to show how the aspect of multi-trips works. Colors black, red and blue in the arcs represent three different vehicles; full arcs represent a trip to attend clients, while dotted arcs represent an idle trip, in which the vehicle does not perform any collection; clients are represented from "1" to "9"; "OD", "$SD1$", "$SD2$" and "FD" are all nodes representing the same depot, which is the only one in the problem.

Figure 7: Multi-trips example



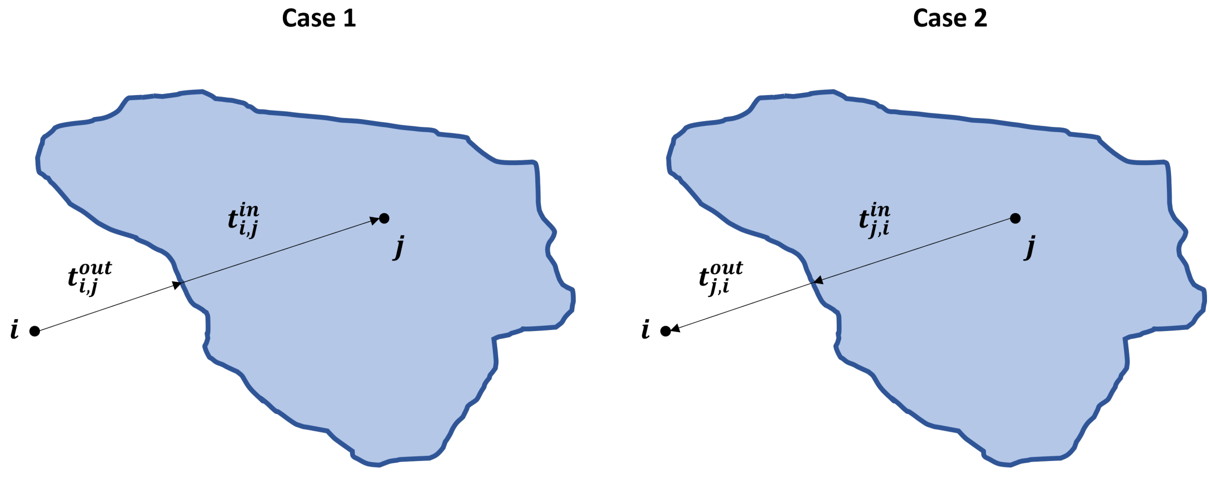
Source: The author

According to the solution in Figure 7, only vehicles whose arcs are colored black and red serve clients. The vehicle with the blue arc, which has no assigned clients, goes from OD directly to the SD nodes and then to FD. The vehicles with black and red arcs exit node OD, attend clients which they are assigned to and comeback to deliver the collected oil to the first sink depot ($SD1$). If only one trip is necessary for each vehicle, then they go from $SD1$ to the Final Depot (FD). If a vehicle needs to complete a second trip, which happens in the case of the vehicle with the black arcs, then it exits $SD1$, collects the oil and brings it to a second sink depot ($SD2$), ending its route by going from $SD2$ to FD. The vehicle with red arcs, which is not required to do a second trip, goes from $SD1$ directly $SD2$ and then to FD.

When a vehicle moves from node i to node j, the distance is defined as $c_{i,j}$. A consequence of the problem at hand being set in an urban context is that $c_{i,j} \neq c_{j,i}$. Therefore, a digraph $G = (N, A)$ is used to represent the problem, where $A = \{(i, j) \in N \times N : i \neq j\}$. The opening and closing hours of each node in $j \in N^{NR}$ are defined by e^{NR} and l^{NR}, respectively. For each client i, service time s_i is also defined. For the route from i to j, the time spent in the route is $t_{i,j}$.

Once the basic VRP problem has been defined, it is possible to include the Access Time Windows restriction, which is related to the TRR. For that, it is easier to use a simple example of how that factor influences the problem. Figure 8 illustrates two cases: in Case 1, a vehicle goes from node i, outside the TRR, to node j, inside the TRR. In Case 2, the vehicle goes from j to i. For both cases, the vehicle needs to cross the boundaries of the TRR, highlighted in blue. The time spent to do that trajectory, in Case 1, is divided in two parts: $t^{out}_{i,j}$, outside the region, and $t^{in}_{i,j}$, inside the region. For Case 2, the times are $t^{out}_{j,i}$ and $t^{in}_{j,i}$. It is worth mentioning that any time spent travelling over the blue boundary is considered to be inside the TRR.

Figure 8: Simplification of the Traffic Restricted Region - Cases 1 and 2



Source: The author

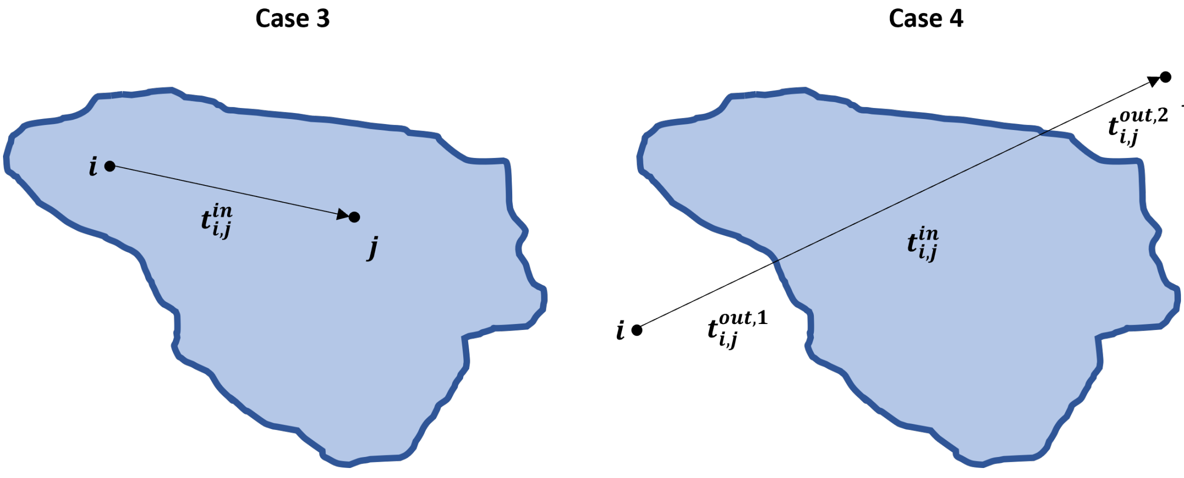
As it was explained in Chapter 3, the TRR imposes a limit of vehicle circulation. Parameter e^{TRR} will be used to represent the opening time of the TRR for the vehicles, while parameter l^{TRR} will be used for its closure. Therefore, all vehicles are prohibited to be inside the TRR either before e^{TRR} or after l^{TRR}.

In Case 1, a variable b_j^k is used to represent the starting time of the service of vehicle k at j coming from node i. Therefore, the time at which the vehicle crossed the border is given by $b_j^k - t_{i,j}^{in}$, that is, the time at which service started at j minus the time spent inside the TRR to reach j. Given the restriction of the TRR, that time of entering the node cannot be lower than e^{TRR} nor higher than l^{TRR}, that is, $e^{TRR} \leq b_j^k - t_{i,j}^{in} \leq l^{TRR}$.

The same logic can be applied to Case 2. The parameter $t_{j,i}^{in}$ represents the time spent inside the TRR when going from node j inside the TRR to node i outside the TRR. In this case, the time of arrival at the border is given by $b_j^k + s_j + t_{j,i}^{in}$, that is, the time at which the service in j started plus the time spent at j collecting used oil (s_j) plus the time spent inside the TRR to reach i. The time of arrival at the border, just as in the case 1, has to happen in the time frame between e^{TRR} and l^{TRR}. Therefore, the inequality in this case is $e^{TRR} \leq b_j^k + s_j + t_{j,i}^{in} \leq l^{TRR}$.

The other two cases are illustrated in Figure 9. In Case 3, the vehicle travels between two nodes inside the TRR without leaving it, while in Case 4 the vehicle goes from one node outside the TRR to another node outside the TRR, but crosses the TRR to do it.

Figure 9: Simplification of the Traffic Restricted Region - Cases 3 and 4



Source: The author

Due to the nature of this problem, Case 3 can only happen for small trucks, which is correctly implemented in the model. Case 4, however, was not restricted for large trucks, although it should. In the model, they are prohibited from attending clients inside the TRR, but not from crossing it. Implementing that restriction would require two different distance matrices, one including the TRR and another blocking any routes that cross it. However, the author did not find a practical way of calculating the second matrix for all nodes which will be used to test the model, since the Google Maps tool (which was used for this purpose) does not have the feature of avoiding a specific area of the map. Another change that would be required for the model to include that case would be to track the vehicles' position and time while they go from one node to another, instead of registering only the times when they start a service, which is what is done in the model. That change would augment the complexity of the model, which would make it difficult for the algorithm to run in a practically feasible amount of time. However, since most real cases of the company's routing include many clients inside the TRR, it is rare for a truck to cross directly the TRR to go from one node in N^{NR} to another node in N^{NR}. Therefore, it was decided not to include the restriction of Case 4 for large trucks in the model, due to its small chance to happen in practice.

Lastly, the model's variables are presented. The binary variable $x_{i,j}^k$ is used to determine if vehicle k travels the arc from node i to node j. If it does, the variable assumes value 1; otherwise, 0. The continuous variable $y_{i,j}^k$ represents the used oil flow carried by vehicle k in the arc $(i, j) \in A$. The variable which indicates the start of the service at client i by vehicle k is b_i^k.

Table 3 summarizes all notation used in the mathematical model.

Table 3: Notation for the mathematical model

Indexes	
$i, j \in \{OD, 1, 2, ..., n, SD1, SD2, ..., FD\}$	The total amount of nodes is $n + \textit{number of SDs} + 2$
$k \in \{1, 2, ..., \lvert K\rvert\}$	$\lvert K\rvert$ is the total amount of vehicles
Sets and subsets	
A	Set of all arcs from node i to j, where $i \neq j$
C	Set of all n clients
SD	Set of sink depot nodes
N	Set of all nodes, formed by $C \cup SD \cup \{OD, FD\}$
N^{TRR}	Subset of clients which are inside the TRR
N^{NR}	Subset of N which contains only nodes outside the TRR
K	Set of all $\lvert K\rvert$ vehicles
K^{LT}	Subset of K formed only by large trucks
Parameters	
D_i	Demand of node i
Q_k	Capacity of vehicle k
VC_k	Fuel consumption per distance rate for vehicle k
T_k	Total allowed duration of a vehicle's workday
$c_{i,j}$	Distance from node i to j
e^{NR}	Opening time of node $j \in N^{NR}$
l^{NR}	Closing time of node $j \in N^{NR}$
e^{TRR}	Opening time of node $j \in N^{TRR}$
l^{TRR}	Closing time of node $j \in N^{TRR}$
s_i	Amount of time to serve node i
$t_{i,j}$	Amount of time to travel the arc from node i to j
$t_{i,j}^{in}$	Time spent inside the TRR when going from node i to node j
M	Sufficiently large positive number
Variables	
$x_{i,j}^k$	Indicates if arc (i, j) is travelled by vehicle k
$y_{i,j}^k$	Amount of used oil carried in arc (i, j) by vehicle k
b_i^k	Beginning of the service in node i by vehicle k

4.2 Objective Function and Constraints

Once the notation has been defined, it is possible to present the mathematical model, which follows a Mixed Integer Linear Programming (MILP) formulation.

Objective Function:

$$Minimize \sum_{k\in K}\sum_{(i,j)\in A} VC_k * c_{i,j} * x^k_{i,j} \tag{4.1}$$

Constraints:

$$\sum_{j\in N\setminus OD} x^k_{OD,j} = 1, \forall k \in K \tag{4.2}$$

$$\sum_{k\in K}\sum_{i\in N} x^k_{i,j} = 1, \forall j \in C \tag{4.3}$$

$$\sum_{i\in C} x^k_{i,j} \leq 1, \forall j \in SD, \forall k \in K \tag{4.4}$$

$$\sum_{i\in N} x^k_{i,j} - \sum_{i\in N} x^k_{j,i} = 0, \forall j \in C \cup SD, \forall k \in K \tag{4.5}$$

$$\sum_{k\in K}\sum_{i\in N} y^k_{j,i} - \sum_{k\in K}\sum_{i\in N} y^k_{i,j} = D_j, \forall j \in C \tag{4.6}$$

$$y^k_{i,j} \leq Q_k * x^k_{i,j}, \forall i \in C, \forall j \in SD, \forall k \in K \tag{4.7}$$

$$y^k_{i,j} \leq M * x^k_{i,j}, \forall (i,j) \in A, \forall k \in K \tag{4.8}$$

$$\sum_{k\in K}\sum_{i\in C}\sum_{j\in SD} y^k_{i,j,k} = \sum_{i\in C} D_i \tag{4.9}$$

$$\sum_{j\in C} x^k_{i,j} \leq (|C|-1)(1-x^k_{OD,i}), \forall k \in K, \forall i \in SD \tag{4.10}$$

$$\sum_{i\in C} x^k_{i,j+1} \leq \sum_{i\in C} x^k_{i,j}, \forall j \in SD \setminus \{|SD|\}, \forall k \in K \tag{4.11}$$

$$x^k_{i,OD} = 0, \forall i \in N \setminus \{OD\}, \forall k \in K \tag{4.12}$$

$$x^k_{i,FD} = 0, \forall i \in C, \forall k \in K \tag{4.13}$$

$$x^k_{FD,j} = 0, \forall j \in N, \forall k \in K \tag{4.14}$$

$$x^k_{i,j} = x^k_{j,i} = 0,\ \forall i \in N, \forall j \in N^{TRR}, \forall k \in K^{LT} \tag{4.15}$$

$$b^k_j \geq b^k_i + s_i + t_{i,j} - M * (1 - x^k_{i,j}),\ \forall (i,j) \in N, \forall k \in K \tag{4.16}$$

$$e^{NR} \leq b^k_j \leq l^{NR} - s_j,\ \ \forall j \in N^{NR},\ \forall k \in K \tag{4.17}$$

$$e^{TRR} + \sum_{i \in N^{NR}} t^{in}_{i,j} * x^k_{i,j} \leq b^k_j \leq l^{TRR} - \sum_{i \in N^{NR}} (s_j + t^{in}_{j,i}) * x^k_{j,i},\ \ \forall j \in N^{TRR}\ \forall k \in K \tag{4.18}$$

$$b^k_{FD} - b^k_{OD} \leq T_k, \forall k \in K \tag{4.19}$$

$$x^k_{i,j} \in \{0,1\}, \forall (i,j) \in A, \forall k \in K \tag{4.20}$$

$$y^k_{i,j} \geq 0, \forall (i,j) \in A, \forall k \in K \tag{4.21}$$

$$b^k_j \geq 0,\ \forall j \in N\ \forall k \in K \tag{4.22}$$

The objective function (4.1) minimizes the vehicles' overall fuel consumption.

Constraints (4.2) enable the start of the process of routing. Although every vehicle is forced to go out of node OD, not all are necessarily put to use. If optimality is reached by using only part of the available fleet, then the unused vehicles go directly from OD to the SD nodes and then to FD.

Constraints (4.3) ensure that every client is visited only once. Constraints (4.4), on the other hand, ensure that every vehicle can visit at maximum once each sink depot.

Constraints (4.5) guarantee the vehicle's flow conservation, because every vehicle that enters a client node or a sink depot node must leave that same node. Constraints (4.6) guarantee that the vehicle visiting client j will collect the demand D_j. The first term on constraints (4.6) indicates the amount of flow in the vehicle when it exits customer j, while the second term indicates the flow when it arrives at j. Therefore, given there is a subtraction between the first and the second term and that D_j is a positive number, then the flow on the exit must be higher than that on the arrival. Constraints (4.7) state that the vehicle k coming from client i can only arrive at any sink depot node if the quantity it carries is lower than or equal to its capacity. This ensures that vehicles will not exceed capacity during the trip. Constraints (4.8) link variables $y^k_{i,j}$ and $x^k_{i,j}$, given that it will only allow $y^k_{i,j} > 0$ in an arc (i, j) if $x^k_{i,j} = 1$ in that arc.

Constraints (4.9) ensure that the sum of all quantities arriving at sink depot nodes must be equal to the sum of all clients' demands. It also explicitly allows the existence of multi-trips, since all collected demand can be taken to any sink depot present in the problem. Constraints (4.10) are used to limit the amount of times a vehicle can leave a sink depot node. Looking at the right-hand side of (4.10), it is possible to see that if $x^k_{OD,i}$ is 1, then both the right- and left-hand sides of the inequality will be equal to 0. For every vehicle k, $x^k_{OD,i}$ is only 1 if that k goes directly from OD to a SD, which means it does not attend clients and it will pass at least one SD before reaching the FD node. Therefore, it is correct that the left-hand side of the inequality is also 0, since that vehicle cannot leave an SD to a client node. However, if $x^k_{OD,i}$ is 0, that is, the vehicle serves clients, then the amount of times the vehicle leaves a sink depot node to attend other clients must be lower than or equal to the amount of customers minus 1, since it already served at least one customer before. If these constraints were not present, then vehicles would be able to go from OD to any SD before serving clients. Although the value of the objective function would not change if that happened, since there is no cost to travel from OD to an SD, it would not be a correct representation of the problem.

Constraints (4.11) ensure that the sink depots are visited by the vehicles in an ascending order. The last SD is included in the constraint in the left-hand side of the inequality, since the index for the variables on that side start at 2 and end at $|SD|$, while on the right-hand side the index goes from 1 to $|SD| - 1$. Just as in the case of (4.10), the value of the objective function would not change if constraints (4.11) were not present. However, by establishing an order among sink depots, these constraints have the purpose of eliminating symmetrical solutions, considering that without them the model wouldn't be able to distinguish different sink depots and the solver could take longer than necessary to solve the instance.

All constraints from (4.12) to (4.15) are used to exclude some variables to reduce the model's dimensionality and to guarantee the functioning of the model. Specifically, (4.12) ensure that no vehicle comes back to OD after starting its journey; (4.13) ensure that vehicles cannot go directly from clients to the FD, as they must pass an SD to unload the oil; (4.14) ensure that vehicles cannot go from FD to any node; and (4.15) set to 0 the variables related to customers which cannot receive large trucks, such as the ones inside the TRR;

Constraints (4.16), (4.17) and (4.18) are related to the access time windows restriction. Constraints (4.16) put the variables b_j^k, if vehicle k goes from node i to node j, on a logical time flow, ensuring temporal consistency. They state that the beginning of the service in j can only be higher or equal to the beginning of the service in i plus the service time in i and the time it takes to travel from i to j. They also disallow subcycles in the problem. Constraints (4.17) limit the variables b_j^k for nodes outside of the TRR to the opening and closing hours of those nodes, guaranteeing that those time windows will be followed.

Constraints (4.18), on the other hand, limit the variables b_j^k to the access time window limits of client j inside the TRR, representing Cases 1 and 2 from Figure 8 in Section 4.1 in the model. All the b_j^k variables related to nodes inside the TRR are limited by e^{TRR} and l^{TRR}, which are the first terms on the right-hand side and left-hand side of the inequality, respectively. The second element of the left-hand side is a sum of elements which will all be 0, except for one at most, in which $x_{i,j}^k$ can be 1. For that case, the left-hand side is $e^{TRR} + t_{i,j}^{in}$, which was explained in Case 1 of Figure 8. That one $x_{i,j}^k$ variable that is not zero represents the entrance of vehicle k in the TRR, since it will come from a node N^{NR} and reaches a node N^{TRR}. Similarly, the last node to be visited in the TRR has its b_j^k variable limited by $l^{TRR} - s_j - t_{j,i}^{in}$, while for the other nodes in the TRR the upper limit of b_j^k is l^{TRR}. Therefore, what happens is that b_j^k for all nodes in the TRR is limited by the access times of the TRR, addressing all arcs such as the one in Case 3 in Figure 9 in Section 4.1. However, the first and last nodes to be visited in the TRR have a more restricted limit of time windows, given by the elements in the summation in each side of the inequality, which implicitly also limits the time windows of all other nodes to be visited in the TRR between the first and the last.

Constraints (4.19) limit the total duration of a vehicle's workday, which cannot be higher than a predefined limit of hours. Finally, constraints (4.20), (4.21) and (4.22) set the domains for the decision variables.

The explanation of the model concludes this chapter. It was a complex task to combine the Access Time Windows attribute of the model with all the other more classical constraints, mainly the Multi-Trip one. The outcome was an original model whose implementations is discussed in the next chapter and that is later validated with different test sets with real data from the partner company.

5 MODEL IMPLEMENTATION

This chapter aims to explain how the model was adapted to a computer program using information provided by the partner company.

5.1 Solver Choice

As previously explained in Chapter 2, the focus of this project is to develop a mathematical model which represents a real problem and solve it using a commercial solver, not a customized heuristic approach. After some research on solvers available for academic purposes, the Gurobi solver was chosen.

Gurobi is a commercial solver for mathematical optimization. Among its solution methods, it applies a state-of-the-art Branch-and-Cut algorithm to solve Mixed Integer Linear Programs (MILP). In the process of solving the problem, Gurobi's algorithm uses four well-known techniques in the MILP field (GUROBI, 2021):

- **Presolve** - reduction of the problem before the Branch-and-Cut procedure. It reduces the amount of possible values for different variables by tightening constraints;
- **Cutting Planes** - they tighten the formulation by removing undesirable fractional solutions during the solution process;
- **Heuristics** - usage of different non-exact approaches to find better values for the objective function in a faster way during the solution;
- **Parallelism** - solving different nodes in the MILP tree search using multiple computer cores.

5.2 Programming Code

The programming language chosen for the implementation was Python 3 (PYTHON, 2021). The choice was based on a series of factors: Gurobi can be called via a library in Python; there are several other libraries for data processing and visualization, such as Pandas and Matplotlib, which were used in the tests; and the author had familiarity with the language, which sped up the implementation process.

In order to get input data for testing, the author requested an Excel file containing client information from the partner company. Table 4 is an example of the provided information:

Table 4: Example of client input data

Node ID	Address	Demand (L)	Service Time (minutes)	Allowed Vehicle Type	Latitude	Longitude
Depot	Depot address	N/A	N/A	All types	Depot Latitude	Depot Longitude
Client 1	Client 1 address	1000	30	Small only	-12.3456	-12.3456
...	...	...	...	...	...	...

In order to keep the clients' confidentiality, the company did not provided their clients names, only a generic ID, which is in the column "Node ID". The author also chose not to reveal the depot's or client 1's address, latitude or longitude to keep confidentiality. Each column's data is briefly explained:

- **Node ID** - ID for identifying each node;
- **Address** - address of the node. It was used to identify the nodes and calculate the distance between all of them using Google Maps;
- **Demand** - the demand column represents the clients' demand for used oil collection in liters. That number was provided having as a basis each clients' typical request for used oil collection. In the case of the depot, demand is not applicable;
- **Service Time** - the service time in minutes for performing the collection in the client. As explained in Chapter 3, it was calculated using a fixed amount of minutes plus an extra period proportional to the amount of oil to be collected, multiplied by a conversion parameter. In the depot's case, the service time is a fixed amount;
- **Allowed Vehicle Type** - that value can be either "All types", when the node does not have a restriction on the vehicle it can receive, or "Small only", when the node can only receive the small vehicle type;

- **Latitude and Longitude** - node's latitude and longitude, which were used to calculate the driving distance between each pair of nodes using Google Maps and to plot the nodes using the NetworkX library in Python (NETWORKX, 2021).

Besides the aforementioned information, more data was provided by the company and researched by the author to implement the model. That information was already presented in Chapter 3, but it is shown again as a reminder:

- **Vehicles' capacity** - There are 3 small trucks and 2 large trucks to attend clients in this problem. Small ones can carry up to 2,500 liters, while large ones can carry up to 5,000 liters;

- **Fleet working hours** - from 7:00 to 19:00.

- **Maximum operation time for a truck** - 8 hours;

- **Time window for truck circulation inside the TRR** - from from 10:00 to 16:00;

- **Diesel consumption for trucks** - The trucks use diesel as fuel. Small and large trucks have a consumption rate of 0.14 L/km and 0.2 L/km, respectively;

- **Average speed** - an average speed was considered during the whole route. For both truck types, the considered speed is 35 km/h;

- **Oil loading time in clients** - the time for loading the truck when attending each client is provided by the partner company, which calculates it by summing the fixed and variable amounts of time;

- **Oil unloading time in the depot** - the time for unloading the truck in the sink depots is fixed and provided by the partner company.

There is an intermediary procedure using the addresses in order to prepare the data for the model. A distance matrix is generated by calculating the distances between each pair of nodes using the Google Maps "Directions" API for Python (GOOGLE, 2021). From that matrix, a time matrix is created by dividing all values in the distance matrix by the average speed.

A third matrix, called TRR distance matrix, is created in the same way as in the first one, although calculating only the distances inside the TRR when a truck crosses the

TRR's border or travels over it. As in the second matrix, a TRR time matrix is created by dividing values in the TRR distance matrix by the average speed.

Using all the mentioned inputs, the program calculates the best possible routing, minimizing the amount of used fuel in a given limited time of processing. Since VRP are known to be complex and hard to achieve the optimal solution, it is not expected that the program should reach the best solution in an acceptable computational time in practice. Instead, a limit of 30 minutes was set for programming the best route for each instance. As explained in Chapter 3, the time it takes to decide the routing of a day is approximately 1 hour, so it was decided together with the company that using half of that time to run the model and get the best possible result would be practically feasible.

The program output is a route for each truck, represented by a series of nodes it should visit. For each travel between nodes, the calculated amount of flow the truck is carrying is also given. The recorded time of the visit in each node is also provided. All of that is represented in a graph as well, in order to get a more visual idea of what is happening in each route.

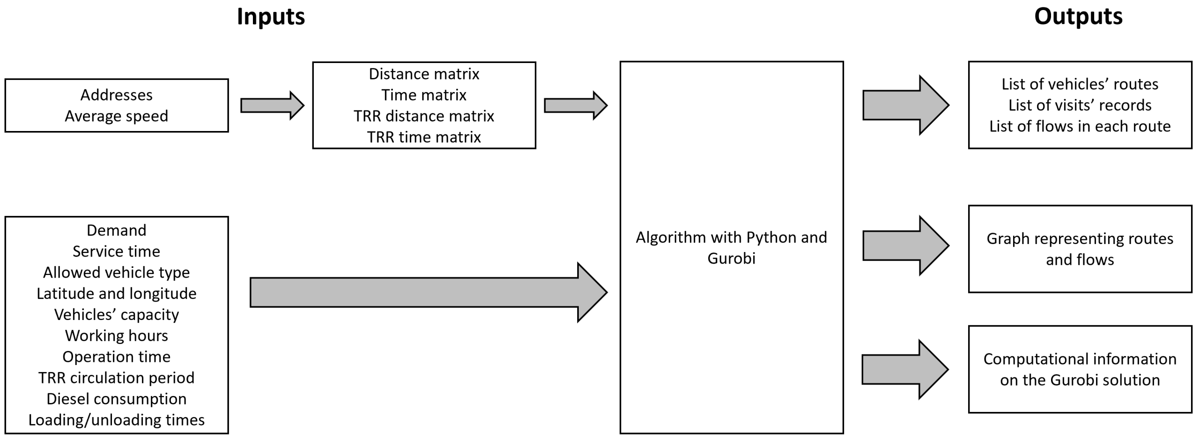
For each instance, there are also computational outputs provided by Gurobi, such as the optimized value for the objective function, computational time, number of variables and constraints, number of explored nodes and the optimality GAP. The GAP formula is the following:

$$GAP = \frac{|\,Upper\ bound - Lower\ bound\,|}{|\,Upper\ bound\,|} \tag{5.1}$$

For the problem in this thesis, the upper bound is the best feasible solution found by the solver, while the lower bound is the best relaxed solution. Note that when the GAP reaches zero, the algorithm has found the optimal solution.

Figure 10 summarizes all inputs and outputs of the program.

Figure 10: Summary of inputs and outputs



Source: The author

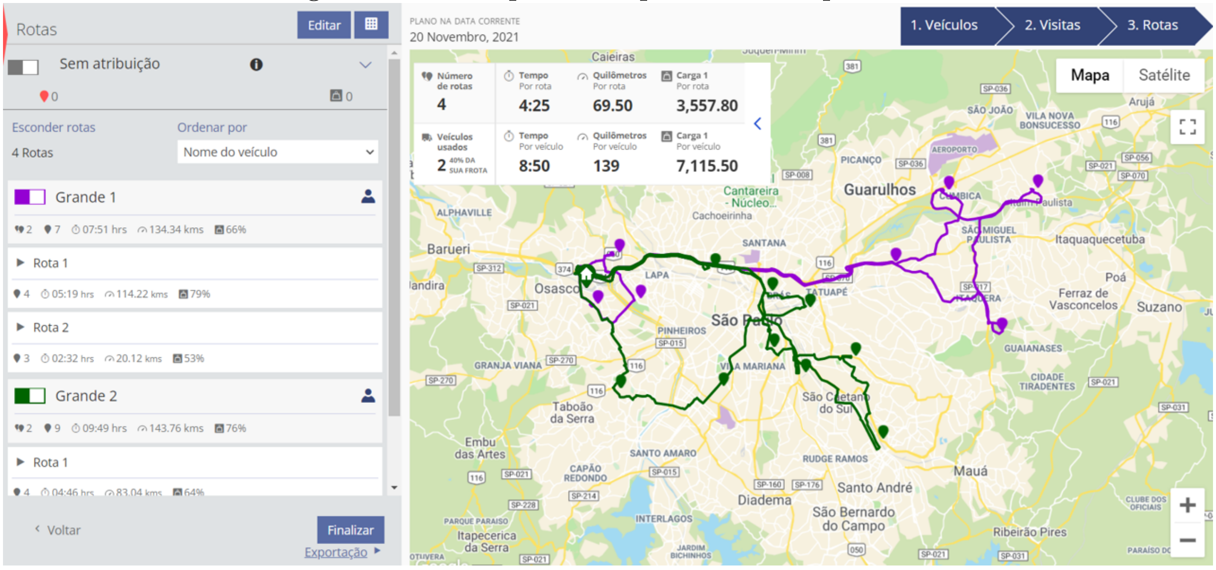
5.3 Comparison with the Currently Used Software

As explained in Chapter 3, the partner company uses the software SimpliRoute to determine each day's routing schedule. In order to compare the model's result and the company's result for a certain instance, the author created a free account in the software, with a trial period of seven days. SimpliRoute's usage is easy and intuitive. First, it is necessary to input data for the vehicles and its characteristics. Then, once they are defined, one can run different instances by changing the clients and their locations. It is also possible to input the limitation of large vehicles attending certain clients, which happens in the problem at hand.

One interesting feature of the software is the possibility to choose from multiple optimization preferences, such as "Minimize number of vehicles", "Balance the load" and "Consider soft time windows". Although there is no option for minimizing overall fuel consumption, "Minimize number of vehicles" solution is the one which comes closest with that objective among all possible options and is the most used one by the partner company. For those reasons the author chose to use "Minimize number of vehicles" as optimization objective to compare SimpliRoute's results with the model's results. However, it is worth noting that while the developed model considers the existence of the TRR and its access time windows, SimpliRoute's software does not.

Figure 11 shows an example of SimpliRoute's solution for an instance with 10 clients and "Minimize number of vehicles" selected.

Figure 11: Example of output from SimpliRoute



Source: SimpliRoute (2021)

On the left there is a scroll menu in which one can visualize each vehicle's assigned clients, total time spent and travelled distance and how much of the capacity was utilized in percentage. On the right, the map shows each vehicle's route with different colors. The pinpoints mark the clients, each one having the color of the assigned truck, and the depot, represented by the pinpoint with a house symbol. On the top left part of the map there is also a summary of some data, such as average time and distance per route and number of vehicles and routes. All the information can be exported to an Excel file. In order to compare the model's solution, which is measured in liters of used fuel, to the one from SimpliRoute, the distance travelled by each truck outputted by SimpliRoute was multiplied by the consumption parameter provided by the partner company.

After explaining how the model works and what outputs are expected, the next chapter encompasses structuring the tests, presenting the results and discussing them. All programs and tests were developed and run using a personal computer with a processor Intel(R) Core(TM) i7-7500U CPU @ 2.70GHz, 8 GB of RAM memory, Windows 10 operational system, Python 3.10.0, Gurobi 9.1 and the default option of core usage by Gurobi, which uses all available cores.

6 COMPUTATIONAL TESTS

This chapter presents the tests of the program in different scenarios by combining a series of factors. It is divided in two sections: the first one tests the program by combining different factors which define a scenario, while the second one tests the program over real-life configurations, which were created together with the company. For each section, the test instances are explained and the results are presented and discussed. At the end, a final section summarizes the most important insights and presents a recommendation for the company.

6.1 Combining Different Factors

The tests in this section use realistic instances based on data and information provided by the. The objective of is to stress the model and to understand how the program behaves under different conditions, by exhausting the combination of three factors.

6.1.1 Test Planning

The scenarios in this section are generated by varying the following factors: number of clients, proportion of clients inside/outside the TRR and number of sink depots.

- **Number of clients** - represents the number of client nodes n in the instance. It was decided to work with multiples of 6. Therefore, the possible number of clients are $n = 6, 12, 18$;
- **Proportion of clients inside vs. outside the TRR** - represents how many clients are inside the TRR and how many are outside of it;
- **Sink depots** - represents how many sink depots there are in the instance. The number of sink depots equals the number of possible trips for each truck. Therefore,

if that number is defined as 2, then each truck is allowed to go out and back to the company's collection centre up to 2 times to unload the collected used oil.

Table 5 provides a summary of the factors that generate the scenarios. The first column presents the factor, while the second column presents its levels. It is worth mentioning that for the "Proportion of clients inside vs. outside the TRR" factor, the levels are presented in proportions of number of clients inside and outside the TRR. So, for example, the level "2/3 vs. 1/3" means that two thirds of the clients are inside the TRR, while one third of them are outside the TRR.

Table 5: Summary of the factors that generate the scenarios

Factor	**Configuration**
Number of clients	6
	12
	18
Proportion of clients inside vs. outside the TRR	1 vs. 0
	2/3 vs. 1/3
	1/2 vs. 1/2
	1/3 vs. 2/3
	0 vs. 1
Number of sink depots	1
	2
	3

Therefore, this testing phase encompasses a total of 45 different tests (3 x 5 x 3), which will be useful to understand how well the chosen model and program perform under different circumstances. For each test, there are plenty of different configurations of clients to be used, since the clients' database provided by the company had more that 200 entries. Therefore, it was decided that for each test configuration, the selected clients would be the ones with the highest demand. The reasoning behind that choice is based on two assumptions: first, that it would be the best way to stress the model, since it would force multiple trips and trucks in most tests; second, it is aligned with the proposition of the case study, which focuses on the key clients of the company, that is, the ones with the highest demands.

6.1.2 Results

Table 6 presents the results using different clients according to the chosen configuration of "Inside vs. Outside the TRR" and varying the number of sink depots. Besides having one column for each factor presented in Table 5, Table 6 presents other columns to describe each instance, briefly explained below:

- **Total Demand (L of used oil)** - total demand of used oil to be collected from the clients in the instance;
- **Test** - identification number of the test;
- **Obj. Function (L of diesel)** - value of the achieved objective function, measured in liters of diesel used as fuel by the trucks to perform the routes. If the instance is infeasible, the cell reads "Infeasible". If the solver did not achieve a solution within the time limit, the cell reads "No solution found";
- **GAP (%)** - the GAP value, as explained in Chapter 5;
- **Used Trucks** - number of trucks used in the solution, differentiated by type;
- **Use of Multi-trips** - "Yes" if at least one truck performed more than one trip; "No" if all trucks performed only one trip;
- **Comp. Time (seconds)** - computational time required by the solver to achieve the solution, in seconds. All tests are limited to 1800 seconds (30 minutes);
- **Integer Variables** - number of integer variables after the pre-solve phase of Gurobi, as explained in Chapter 5;
- **Constraints** - number of constraints after the pre-solve phase of Gurobi, as explained in Chapter 5.

If the solution is infeasible or is not found under the time limit, some values of the table for that instance are set as "N/A", which means "Not Applicable". Results are shown in Table 6.

Table 6: Tests with different configurations

Clients	In vs. Out the TRR	Total Demand (L of used oil)	Sink Depots	Test	Obj.Function (L of diesel)	GAP (%)	Used Trucks	Use of Multi-Trips	Comp. Time (seconds)	Integer Variables	Constraints
6	1 - 0	8130	1	1	Infeasible	N/A	N/A	N/A	0.05	90	187
			2	2	23.75	0.00	3 small	Yes	0.89	195	433
			3	3	23.75	0.00	3 small	Yes	0.96	243	520
	2/3 - 1/3	9951	1	4	38.64	0.00	3 small, 1 large	No	0.36	139	322
			2	5	38.55	0.00	3 small, 1 large	Yes	4.17	229	510
			3	6	38.55	0.00	3 small, 1 large	Yes	6.47	293	633
	1/2 - 1/2	9963	1	7	39.04	0.00	3 small, 1 large	No	0.64	155	346
			2	8	38.95	0.00	3 small, 1 large	Yes	3.75	247	540
			3	9	38.95	0.00	3 small, 1 large	Yes	7.54	315	671
	1/3 - 2-3	9918	1	10	37.92	0.00	3 small, 2 large	No	0.95	171	372
			2	11	36.31	0.00	3 small, 1 large	Yes	4.95	269	583
			3	12	36.31	0.00	3 small, 1 large	Yes	10.31	341	722
	0 - 1	8801	1	13	31.52	0.00	1 small, 2 large	No	0.72	360	458
			2	14	31.52	0.00	1 small, 2 large	No	2.25	325	688
			3	15	31.52	0.00	1 small, 2 large	No	5.76	401	892
12	1 - 0	14680	1	16	Infeasible	N/ A	N/A	N/A	0.11	471	1039
			2	17	No solution found	N/A	N/A	N/A	1800.00	591	1273
			3	18	41.10	8.87	3 small	Yes	1800.00	675	1432
	2/3 - 1/3	16998	1	19	Infeasible	N/ A	N/A	N/A	7.26	513	1055
			2	20	50.58	3.20	3 small, 1 large	Yes	1800.00	587	1238
			3	21	50.58	6.70	3 small, 1 large	Yes	1800.00	778	1767
	1/2 - 1/2	16931	1	22	Infeasible	N/A	N/A	N/A	22.23	557	1190
			2	23	51.89	9.69	3 small, 2 large	Yes	1800.00	839	1742
			3	24	51.89	11.14	3 small, 2 large	Yes	1800.00	939	1924
	1/3 - 2-3	16584	1	25	45.30	0.00	3 small, 2 large	No	15.01	617	1231
			2	26	45.30	0.00	3 small, 2 large	No	630.00	796	1638
			3	27	45.30	0.00	3 small, 2 large	No	918.00	919	1870
	0 - 1	14490	1	28	48.83	0.00	3 small, 2 large	No	58.67	785	1535
			2	29	48.83	7.07	3 small, 2 large	No	1800.00	985	1985
			3	30	48.83	12.15	3 small, 2 large	No	1800.00	1098	2446
18	1 - 0	19563	1	31	Infeasible	N/A	N/A	N/A	0.14	1029	2146
			2	32	Infeasible	N/A	N/A	N/A	0.22	1203	2542
			3	33	No solution found	N/A	N/A	N/A	1800.00	1323	2773
	2/3 - 1/3	9621	1	34	Infeasible	N/A	N/A	N/A	435.00	1115	2344
			2	35	38.26	20.21	3 small	Yes	1800.00	1333	2770
			3	36	38.26	20.34	3 small	Yes	1800.00	1560	3186
	1/2 - 1/2	23270	1	37	Infeasible	N/A	N/A	N/A	0.17	1211	2451
			2	38	63.66	8.82	3 small, 2 large	Yes	1800.00	1447	2985
			3	39	64.43	12.10	3 small, 2 large	Yes	1800.00	1616	3303
	1/3 - 2-3	22620	1	40	Infeasible	N/A	N/A	N/A	0.17	2100	2701
			2	41	71.68	11.79	3 small, 2 large	Yes	1800.00	1597	3277
			3	42	72.18	16.24	3 small, 2 large	Yes	1800.00	1778	3618
	0 - 1	18343	1	43	Infeasible	N/A	N/A	N/A	0.38	1715	3407
			2	44	63.65	29.67	2 small, 2 large	Yes	1800.00	2005	4067
			3	45	No solution found	N/A	N/A	N/A	1800.00	2210	4457

6.1.3 Discussion of the Results

There are plenty insights to be discussed from the tests' results. First, it is interesting to notice that for many configurations of clients, the model was able to find the same optimal solution for two or three different instances, such as in in Tests 2 and 3 and Tests 13, 14 and 15, respectively. This happens when increasing the number of sink depots does not change the answer, since the best answer has been found. Since it is known that the small trucks have the lowest fuel consumption per distance and the solution for Tests 2 and 3 uses all small trucks, then it cannot be improved. For Tests 13, 14 and 15, although the solution uses large trucks, increasing the number of sink depots did not change the solution.

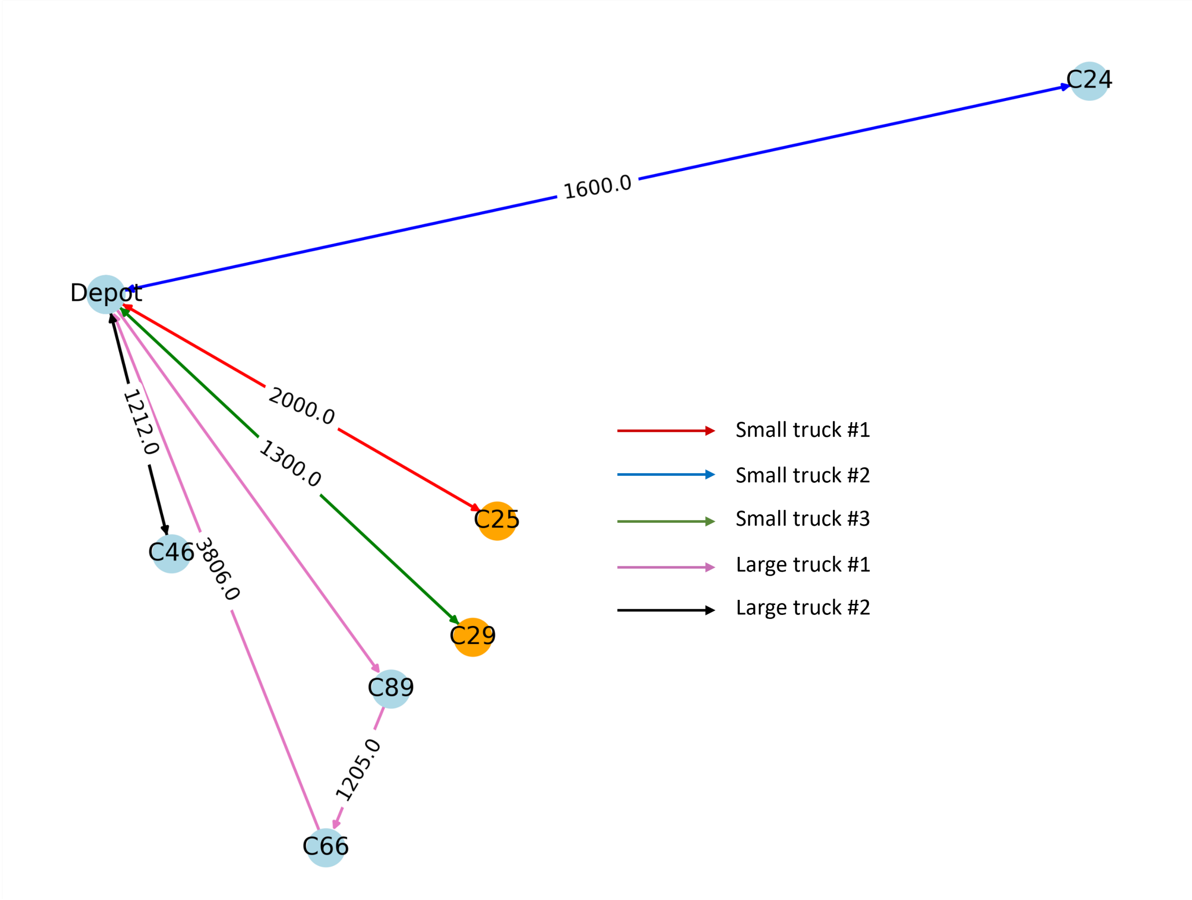
Regarding each factor's influence over the problem's complexity, it is evident that increasing the number of clients is the factor that has the greatest impact over that. One can infer the complexity from the increase in both the number of integer variables and number of constraints. The change in number of clients alters those two columns' values more than changes in the configurations of clients or number of sink depots.

A third remark should be pointed to the Infeasible instances in the table. Tests 1, 16, 19, 22, 31, 32, 34, 37, 40, 43 are infeasible because in those scenarios the amount of multiple trips allowed is not enough for the small trucks to attend all clients inside the TRR without violating their capacity constraints. For tests with 6 clients, it only happens in Test 1, in which all clients are inside the TRR. However, once the number of clients increases, there are tests that have clients outside the TRR that are still infeasible, such as Test 16, 19 and 22 and the other infeasible tests in scenarios with 18 clients.

An interesting result brought by the usage of multi trips can be observed in Tests 5, 8 and 11. The algorithm found a new optimal solution when allowed to run multi-trips, which can be observed when comparing the Objective Function value from those tests to the ones before them. For Tests 39 and 42, however, increasing the amount of sink depots from 2 to 3 actually worsened the objective function. What happened in those cases is that increasing the amount of allowed multi-trips also increased the complexity of the problem to be solved and, due to the established computational time limit of 30 minutes, the model was not able to reach a solution as good as the one in the 2 sink depots scenarios.

As mentioned in Section 5.2, the author used a Python library to visualize the routing results. Figure 12 represents the routing of the optimal solution for Test 10. The nodes outside the TRR are colored in blue, while the ones inside it are in orange. Each edge color represents a truck, as shown in the legend, while the number over the edge shows the amount of collected used oil the truck is carrying when travelling that edge, except when it is not carrying anything yet, in which case there is no number.

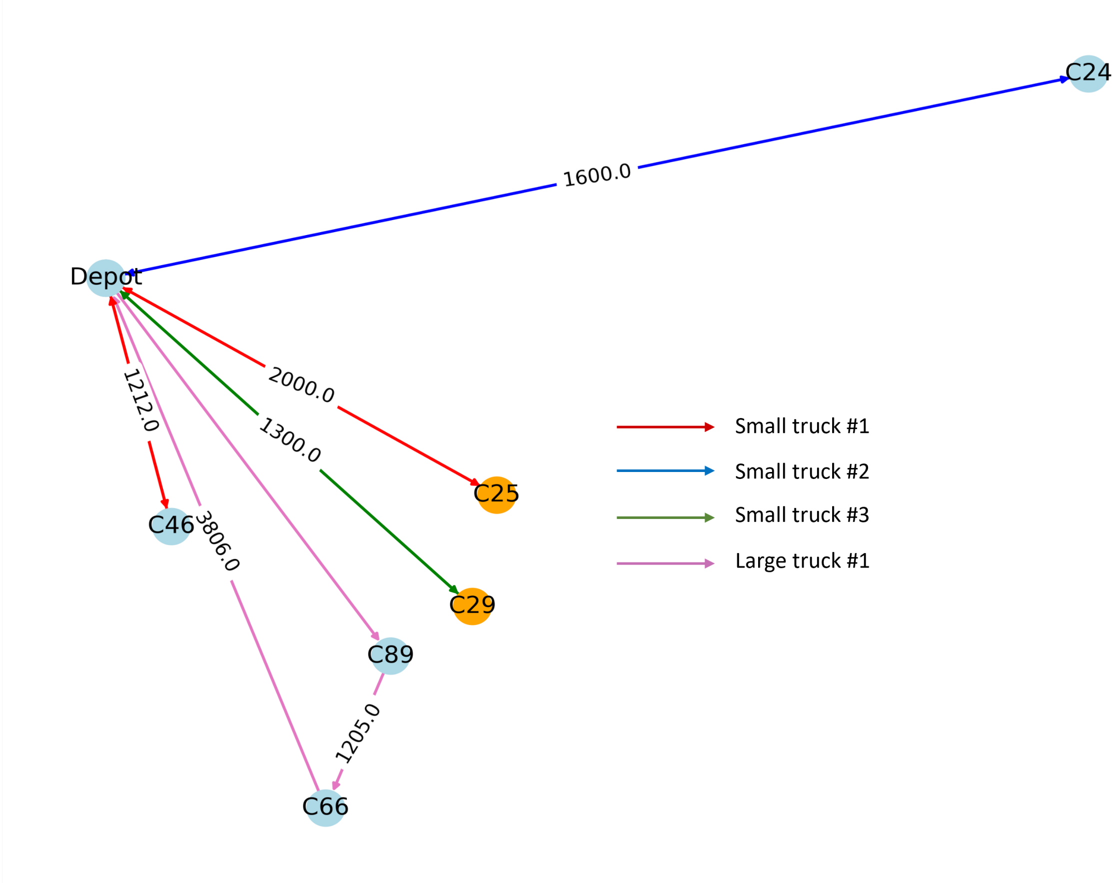
Figure 12: Visualization of the solution of Test 10



Source: The author

In this solution, all trucks go back and forth to collect a single client's demand, except for Large truck #1, which collects used oil both from C66 and C89. As it is possible to see, only small trucks enter the TRR to collect used oil, which shows the model is adhering to the TRR constraints correctly. Figure 13 represents the optimal solution for Test 11 and is shown to compare both cases.

Figure 13: Visualization of the solution of Test 11



Source: The author

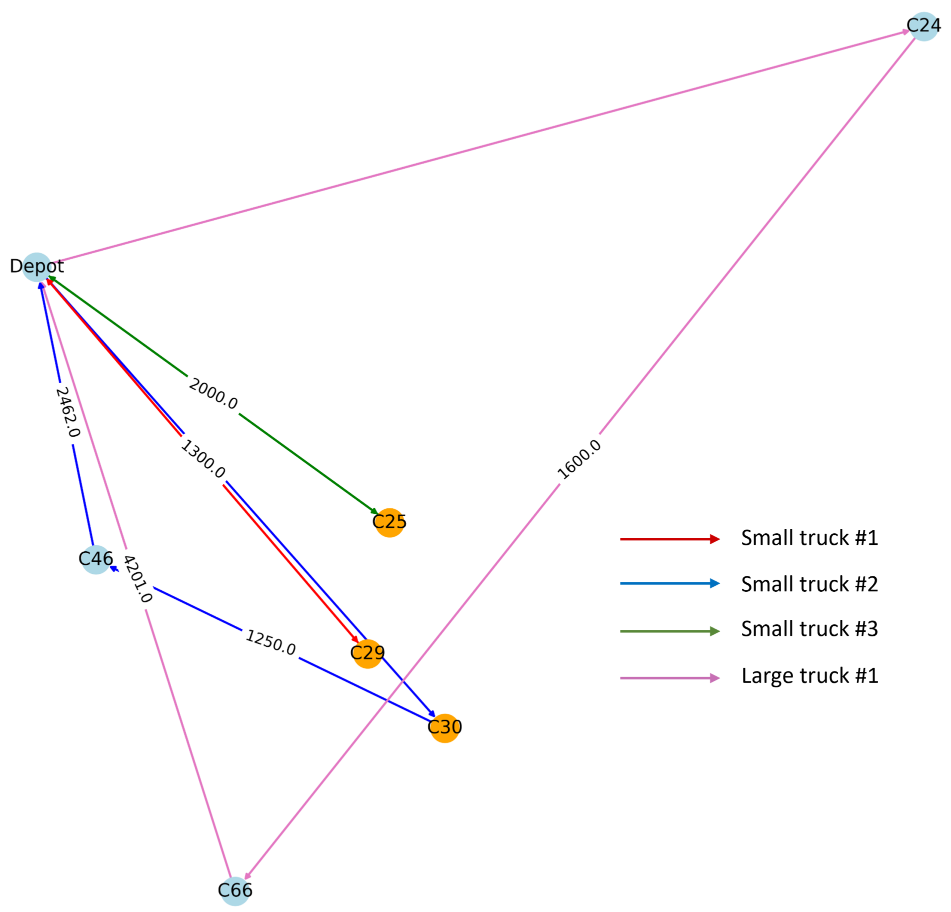
The only difference between Tests 10 and 11 is that in the latter multi-trips are allowed. That explains the only change made by the algorithm from one to another, which is substituting Large truck #2 for Small truck #1, which performs two trips. That reduces the objective function of total diesel used as fuel, since the small trucks' consumption rate of diesel is lower. That change also happens in Tests 5 and 8, in which increasing the amount sink depots reduced the value of the objective function.

When analysing tests with 12 and 18 clients, the weight of the instances' complexity becomes more clear. When the amount of clients increases, some instances cannot output a feasible solution within the predefined computational time limit of 30 minutes. Particularly, Tests 17 and 33 do not find a solution because they are the most constrained instances, since all clients are inside the TRR. For example, for Test 17 the average amount to be collected by each small truck in each trip, if using both available trips in that instance, is $14680 \div 3 \div 2 = 2446$, which is almost the small truck capacity of 2500.

Similarly, for Test 33 the average amount is $19563 \div 3 \div 3 = 2173$, also close to 2500.

The model's limitation related to large trucks crossing the TRR, as introduced in Case 4 of Figure 9 in Section 4.1, becomes evident in some instances. Although the model blocks large trucks from visiting clients inside the TRR, it was not devised to prohibit them from crossing the TRR to go from one N^{NR} node to another N^{NR} node. However, in most tests that does not happen, because the TRR shape is relatively compact, with four out of the five configurations of clients inside and outside the TRR having at least one client inside the TRR. Therefore, the optimal solution in most cases is that a large truck will attend clients in one part of the city without crossing it and, therefore, not crossing the TRR, while small trucks attend clients inside the TRR and sometimes also outside of it. However, there are some few cases in which a large truck crosses the TRR. The optimal solution for Test 7, illustrated in Figure 14 is an example of one of those cases.

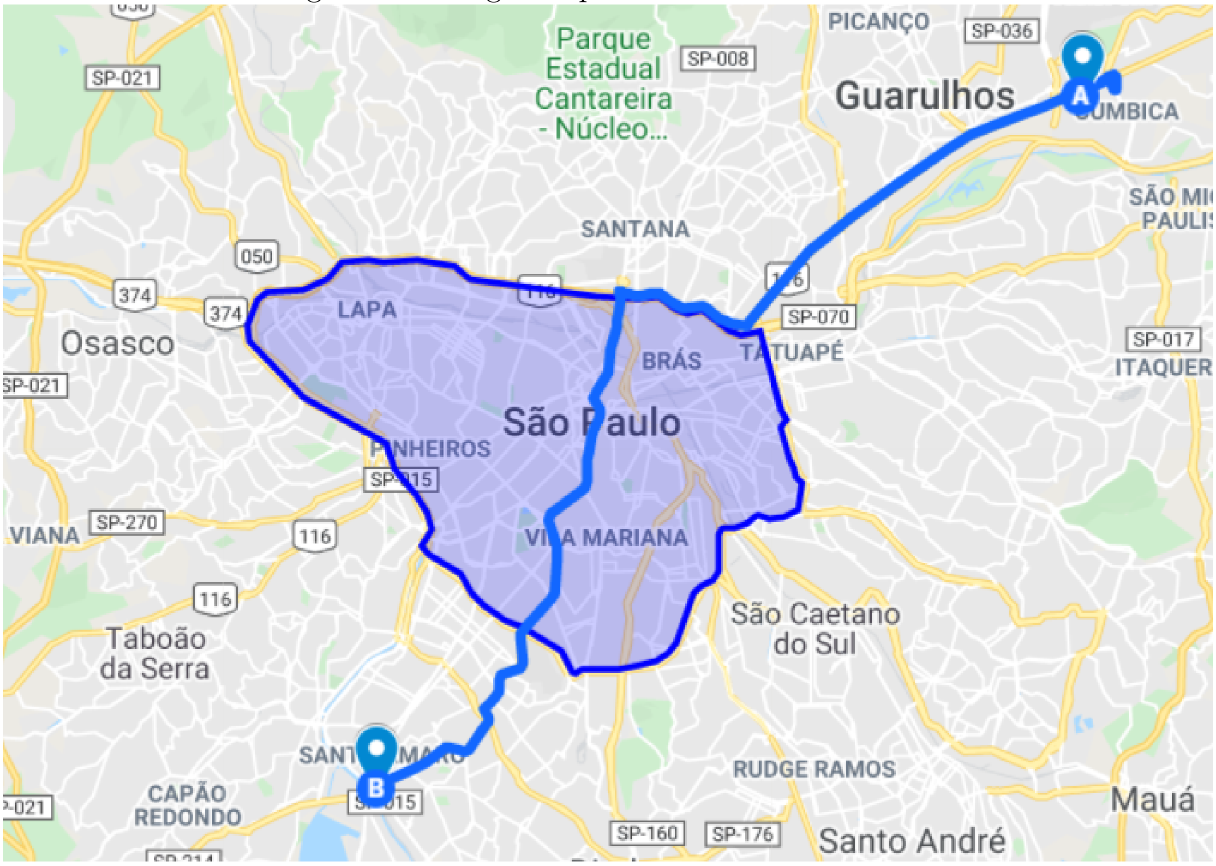
Figure 14: Visualization of the solution of Test 7



Source: The author

As it is possible to see, Large truck #1 goes from C24 to C66. By calculating that route usin Google Maps, which was also used to get the distances between all clients, the output is the following, shown in Figure 15.

Figure 15: Google Maps' route from C24 to C66



Source: The author

It becomes clear in Figure 15 that solutions in which a large truck needs to cross the TRR to attend different clients might not be possible in real life. However, since that did not happen in most tests, it is safe to assume that the model could be used for most routing cases, although its output should be reviewed before being put to use in order to guarantee that no TRR restrictions are being violated. This evidences the fact that the model can still be improved to prevent situations such as that one from occurring, although that would demand additional research by the author.

6.2 Tests with Real-life Configurations

This section is dedicated to testing the model by using real-life instances of clients. The instances were devised together with people from the partner company, in order to guarantee that they made sense from a real-life point of view. According to them, daily amounts of clients to be visited ranges from 20 to 27 and most cases have the "Inside vs. Outside the TRR" configuration with a value around "1/3 vs. 2/3". That happens mainly for two reasons: because most of their clients are outside the TRR and because the routing staff purposefully does not set too many clients inside the TRR to be attended in the same day. As for the number of sink depots, it was set to 2. The main reason for that is that, according to the company, most cases do not exceed 2 trips per truck.

As explained in Chapter 5, the company's results used for comparison were obtained by running the instances in the software SimpliRoute with the "Minimize number of vehicles" optimization, which is currently used by the company. Since the amount of daily clients has a range from 20 to 27, it was decided to perform 4 tests with 21 clients and 4 tests with 24. Each test has a different pool of clients, which was randomly generated and the validated with the partner company in terms of accordance with real-life situations. Consequently, the total demand is also different for each test.

6.2.1 Results

Differently from the tests in the previous section, these tests have some fixed parameters, which are the proportion of clients inside and outside the TRR and the number of sink depots. Therefore, the results table for this section will not have a column for those parameters. The column recording if multi-trips were used was also taken out, because all tests used it. The column in which computational time is registered was also excluded because all instances reached the time limit of 30 minutes. Columns that recorded number of integer variables and constraints were also removed from this table, since the objective in this section is not to analyze the model in detail, but rather to compare its solutions with the ones from SimpliRoute. There are also extra columns for the solution of SimpliRoute, which will be indicated by "SR" and the label of the column, and columns for comparing the results from the two sources. The new columns are the following:

- **SR Fuel Consumption (L of diesel)** - fuel consumption in SimpliRoute's solution, calculated by multiplying each vehicle's total travelled distance by the parameter of fuel consumption rate provided by the company. The parameters are the same as the ones used in the model;

- **SR Used Trucks** - number of trucks used in SimpliRoute's solution, differentiated by type;

- **Difference in Fuel Consumption (L of diesel)** - calculates SimpliRoute's fuel consumption minus the model's objective function value, which also measures fuel consumption. The value is measured in liters of diesel. If the value is positive, it means that SimpliRoute's fuel consumption is higher; if it is negative, then the developed model's solution consumes more fuel;

- **Fuel Savings (%)** - divides the difference in fuel consumption by the fuel consumption in SimpliRoute's solution. It represents how much the fuel savings mean in terms of what is currently consumed by the company.

Results are shown in Table 7.

Table 7: Tests with real-life configurations

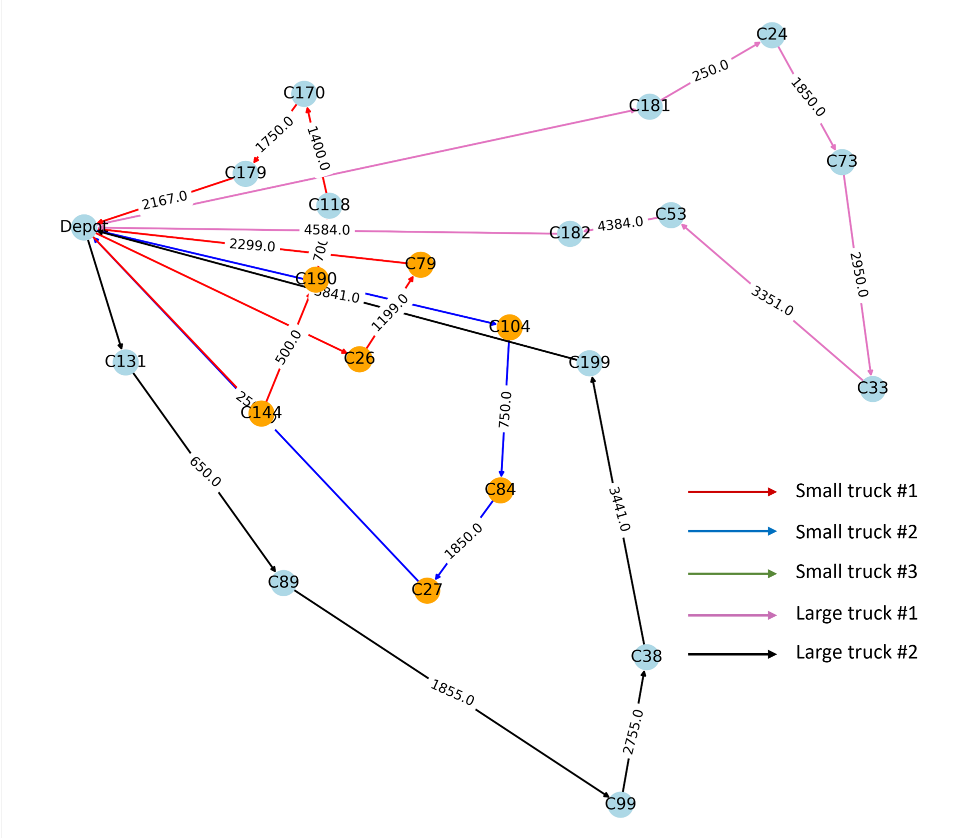
Clients	Test	Total Demand (L of used oil from clients)	Results from the Model			Results from SimpliRoute		Comparison	
			Obj.Function (L of diesel)	GAP (%)	Used Trucks	SR Obj. Function (L of diesel)	SR Used Trucks	Difference in Fuel Consumption (L of diesel)	Fuel Savings (%)
21	1	15663	54.37	37.10	3 small, 1 large	65.37	1 small, 2 large	11.00	16.83
	2	10995	42.61	29.22	3 small, 1 large	53.92	1 small, 2 large	11.31	20.98
	3	15391	58.29	30.40	2 small, 2 large	65.89	1 small, 2 large	7.60	11.53
	4	18631	63.51	29.63	3 small, 2 large	69.80	1 small, 2 large	6.29	9.01
24	5	17951	66.74	39.06	2 small, 2 large	65.17	1 small, 2 large	-1.57	-2.41
	6	23412	77.55	35.71	3 small, 2 large	76.09	1 small, 2 large	-1.46	-1.92
	7	20805	73.75	31.91	3 small, 2 large	81.30	2 small, 2 large	7.55	9.29
	8	21768	64.96	26.89	3 small, 2 large	59.88	1 small, 2 large	-5.08	-8.48

6.2.2 Discussion of the Results

The most interesting value to observe in each instance is the difference in the fuel consumption between the model and SimpliRoute and the fuel savings in percentage terms. Out of the 8 tested instances, 5 had a positive value in the Difference column, which means that in 5 tests the developed model performed better in terms of reducing the amount of consumed fuel when compared to SimpliRoute's solution. Particularly, for Tests 46 to 49, which have 21 clients, all values were positive. Observing the savings in terms of percentage on the last column, it is possible to see that all of them were meaningful, over 9%. Regarding Tests 50 to 53, which had 24 clients, the developed model performed better in 1 out of 4, although in Tests 50 and 51 the difference was small, of only 2%. In Test 52, the model again performed better with savings of 9%.

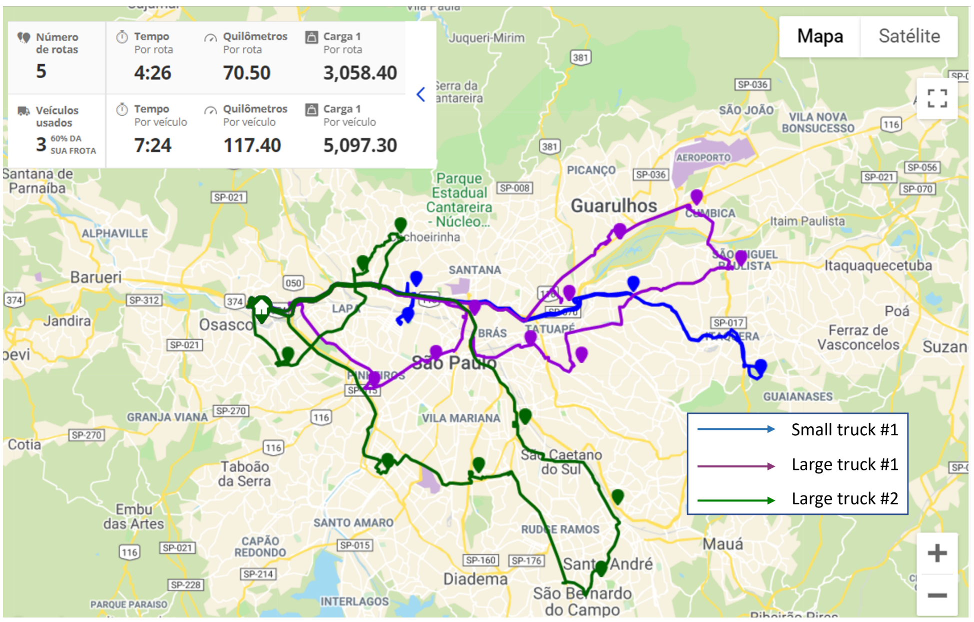
Although those numbers already show that the model has a better performance than SimpliRoute in some cases, it is also interesting to look at the generated routes. For example, Test 48 is illustrated in two Figures: Figure 16 illustrates the solution from the model, while Figure 17 illustrates the solution from SimpliRoute.

Figure 16: Visualization of the solution of Test 48 - model's solution



Source: The author

Figure 17: Visualization of the solution of Test 48 - SimpliRoute's solution



Source: SimpliRoute (2021)

It is interesting to notice how the developed model uses more trucks and balances the load more evenly across them. Although that was to be expected, since SimpliRoute's selected optimization choice was “Minimize number of vehicles”, it evidences yet another advantage of the model's solution. Another visible difference between the solutions is that the model gives preference to small trucks, given that their fuel consumption rate is lower. While there are three trips performed with small trucks and two with large trucks in the model's solution, SimpliRoute's solution uses one trip with a small truck and four trips with large trucks. However, both solutions have the issue of at least one large truck crossing the TRR, although that happens once in the model's solution and twice in Simpli Route's solution.

6.3 Main Insights and Managerial Recommendations

In the first section of testing, it is possible to observe that the model ran well for instances with a low number of clients, such as the ones with 6 clients, achieving the optimal solution in a few seconds. As the number of clients increased, the GAPs increased a bit, reaching a maximum of 30% in Test 41. It must be noted, however, that the model was unable to reach a solution for particular instances, mainly the ones in which small trucks were almost at full capacity.

The most interesting results appeared when increasing the number of sink depots, which allowed multi-trips. For some cases, such as Tests 5, 8 and 11, the model prioritized using small trucks in multiple trips instead of using the large ones, a decision that is not trivial to make, since in a real case one would have the urge to use all available trucks. However, to minimize the overall fuel consumption, which is the objective of this model, that would be the preferred choice.

In the second section of testing, the difference in performance between the currently used routing software, SimpliRoute, and the model becomes evident. For a low amount of clients in the instance, the model performed much better, with an improvement of fuel savings ranging from 9 to 21%. For instances with more clients, the performance of the model decreased and its solutions were a bit worse for some instances.

Overall, it can be said that using the developed model can decrease the amount of consumed fuel for the company's fleet, making the operation more sustainable. Although not all outputted solutions are better than the SimpliRoute's solutions, if the company runs both algorithms, it would be able to choose one or another, depending on which result is better, and even combine them, if convenient.

Another remark should be made regarding the process of reading the outputted routes and adapting them. According to the partner company, SimpliRoute's solutions do not consider the restrictions of Access Time Windows. The process of creating the routes require the staff to read the outputs and evaluate if any real-life restrictions are being violated and, if necessary, adapt the routes in order to fix any issues. This is another positive addition that the usage of the model would bring to the company, since it would eliminate a great part of this extra work of having to manually adapt routes in order not to violate TRR time restrictions. Therefore, it could be recommended for the company to include the model in its daily routing procedure and to compare the results between both algorithms, so that the decision makers of the partner company can make better choices of routes.

In order to evaluate the model's performance in the real-life configurations from a practical point of view, the results were presented to an employee of the partner company that is involved in the daily routing procedure. Similarly as done in this thesis, they were presented along with the results from SimpliRoute for each instance. The employee was very interested in the results and it was claimed that all of the model's results in real-life configurations made sense and could be effectively put into practice. He was also interested in further developing the model so it could be run in a lower amount of time, so that it could be easier to use in daily procedures.

7 CONCLUSION AND FUTURE RESEARCH

7.1 Thesis Summary

This thesis studied a logistics problem of a company responsible for collecting used oil from clients all over Brazil. Particularly, the author was concerned with a limited fleet operating in the Metropolitan Region of São Paulo, which is responsible for collecting used oil and bringing it back to a collection centre. The main objectives were to interpret that real-life situation, adapt it into a mathematical model using VRP theory, and solve the originated problem by using the Python language and Gurobi as a commercial solver. From there, the author was able to derive insights and solutions which can be used to improve the company's logistics by reducing the amount of consumed fuel in the trucks, which by consequence also reduces logistics costs and steers the company towards a more sustainable operation.

In order to achieve those objectives, a literature review was performed for the themes of Circular Economy and Reverse Logistics, in order to better comprehend the company's activities and how it contributes to reducing the total amount of lubricant used in the industry by applying concepts from those fields of knowledge. A review was also performed for VRP studies. In order to translate the company's problem to a mathematical language, the author needed to get acquainted with all VRP variants which would be necessary to do that. Therefore, the CVRP and variants such as the VRP with Time Windows, with Heterogeneous Fixed Fleet, with Site Dependencies, with Multi-Trips and with Access Time Windows needed to be covered in the review. Besides, a review on Rich VRP was also performed, since the addressed problem falls into that category.

By using concepts present in studies from all of those fields, and by collecting all relevant information to precisely describe the problem at hand, the author developed an original model for a Time-Constrained Site-Dependent Vehicle Routing Problem with Heterogeneous Fixed Fleet, Access Time Windows and Multi-Trips, whose purpose is to solve the partner company's problem of used oil collection.

From a theoretical point of view, the most important outcome is the unique combination of constraints in the developed VRP model. The features of Multi-Trips and Access Time Windows were modeled in an unprecedented way in this work. The author was not able to find in the literature a model which implements those aspects in the same way as it was done in this thesis. Modeling the multi-trips to be solved by an exact approach was a challenging task, since most authors prefer to apply heuristics or other approximate methods to solve VRP with multiple trips. The usage of artificial nodes such as the ones of the sink depots was the preferred option to solve that. Access time windows examples for the restricted region were also scarce in the literature. The preferred choice was to use a restriction to impose a time window which guarantees that no vehicle will trespass the time window of the restricted region. Combining both restrictions along with the others present in the problem resulted in an original model, not previously available in literature.

The model's implementation was done using Python and Gurobi along with real data provided by the company, although no names were revealed, keeping confidentiality. The first set of tests aimed at stressing the model and evaluating its performance by changing different variables. Results showed that the model performed very well for instances with amounts of clients lower or equal to 18, but it is evident that it was the variable that most influenced computational times. The possibility to use multi-trips in some instances created interesting solutions, which changed if compared to solutions where only one trip was possible per truck, providing better results.

The second set of tests aimed at comparing the model's results with the ones from SimpliRoute, the software currently used by the company for routing. Results from that section demonstrated that the model is able to generate solutions which consume less truck fuel in the routing, which reduces logistics costs and makes the operation more sustainable. From a managerial and practical perspective, the main outcome of this thesis was to create a model that outperforms the company's software in some scenarios. Besides, it considers the Access Time Windows, a restriction which is ignored in the algorithm embedded in SimpliRoute. From that point of view, the work here developed can contribute to the company's operation, which could combine results from both algorithms or choose the best one for each instance.

7.2 Limitations and Future Research

The main limitations of this thesis concern the developed mathematical model and the approach used to solve it. As explained in Chapters 3 and 4, the most complex characteristics of the problem treated in this thesis are the Multi-Trips and the Access Time Windows restrictions. Although the former has been thoroughly studied in previous literature, the latter is a recent addition to the VRP class. Therefore, combining both was the greatest challenge in the modeling part of this work and even though the presented model provides a decent approximation to the reality of the company's operation, it is not a perfect representation of reality in all possible cases.

The main limitation of the model is that it does not provide an accurate representation for cases in which large trucks can cross the TRR. Although that should not happen, the model is only able to limit the access of large trucks to clients inside the TRR, but it does not prohibit their passage through that area if they need to travel between two nodes outside the region. The reason behind that limitation is that modeling that prohibition would require a much more complex model, which should accompany each truck's position all times and use two distance and time matrices, one which considers the TRR and another which would "eliminate" it from the map, prohibiting vehicles to pass through it. This last requirement would be particularly hard to model because the author is unaware of any softwares that can calculate the minimum distance between a set of points that would perform that elimination of a part of the map. Another inaccuracy caused by the lack of that restriction is that large trucks are able to transit over the border of the TRR outside of its access time windows, even though they should only be able to do it within that access time window. Therefore, a future development of this work could be to focus on adapting the mathematical model to better represent the reality by including those particular restrictions.

The second important limitation concerns the chosen approach to solve the VRP. The exact approach using a commercial solver was the choice of the author because it is an easy and intuitive way of solving an optimization problem. It also fits this thesis because the focus of the author was more on the innovation of the mathematical model rather than on the method to solve the problem. However, VRP are known to be difficult to solve, which is why the vast majority of studies in that field has the objective of developing new solution methods using heuristics, metaheuristics, matheuristics and machine learning. For that reason, another next step of this work would be to take a different approach to solve the problem and use one of the many available methods to solve VRP, or to develop a new one.

7.3 Final Considerations

From a pedagogical and personal point of view, the development of this thesis granted the author the opportunity to apply various skills and to acquire theoretical and practical knowledge from different areas, among which:

1. Mathematical modeling of an optimization problem;
2. Computational implementation of a model using Python and Gurobi;
3. Application of Reverse Logistics and Circular Economy in the industry of used oil collection;
4. Development of a test methodology and gathering insights from the results;
5. Development of a complex academic work.

With this study, the author hopes to bring light to the importance of used oil collection in Brazil and to contribute to the connection between theory and practice in VRP studies, creating a basis for future work that will be developed regarding problems similar to the one covered in this thesis. Hopefully the partner company will be able to improve its logistics operation and reduce fuel consumption, steering towards a more sustainable way of working.

REFERENCES

ABNT. *Norma ABNT NBR 10004:2004*. 2021. Accessed: 2021-06-01. Available at: ⟨https://www.abntcatalogo.com.br/norma.aspx?ID=936⟩.

ALONSO, F.; ALVAREZ, M. J.; BEASLEY, J. E. A tabu search algorithm for the periodic vehicle routing problem with multiple vehicle trips and accessibility restrictions. *Journal of the Operational Research Society*, Springer, v. 59, n. 7, p. 963–976, 2008.

ANP. *Óleos Lubrificantes Usados ou Contaminados (OLUC)*. 2021. Accessed: 2021-03-18. Available at: ⟨https://sinir.gov.br⟩.

ARCHETTI, C.; SPERANZA, M. G. A survey on matheuristics for routing problems. *EURO Journal on Computational Optimization*, Springer, v. 2, n. 4, p. 223–246, 2014.

BALDACCI, R.; MINGOZZI, A.; ROBERTI, R. New route relaxation and pricing strategies for the vehicle routing problem. *Operations Research*, INFORMS, v. 59, n. 5, p. 1269–1283, 2011.

BALDACCI, R.; TOTH, P.; VIGO, D. Exact algorithms for routing problems under vehicle capacity constraints. *Annals of Operations Research*, Springer, v. 175, n. 1, p. 213–245, 2010.

BATTARRA, M.; CORDEAU, J.-F.; IORI, M. Chapter 6: Pickup-and-delivery problems for goods transportation. In: *Vehicle Routing: Problems, Methods, and Applications, Second Edition*. [S.l.]: SIAM, 2014. p. 161–191.

BENT, R.; HENTENRYCK, P. V. A two-stage hybrid local search for the vehicle routing problem with time windows. *Transportation Science*, INFORMS, v. 38, n. 4, p. 515–530, 2004.

BRAEKERS, K.; RAMAEKERS, K.; NIEUWENHUYSE, I. V. The vehicle routing problem: State of the art classification and review. *Computers & Industrial Engineering*, Elsevier, v. 99, p. 300–313, 2016.

BRANDÃO, J. A tabu search algorithm for the heterogeneous fixed fleet vehicle routing problem. *Computers & Operations Research*, Elsevier, v. 38, n. 1, p. 140–151, 2011.

BRANDAO, J.; MERCER, A. A tabu search algorithm for the multi-trip vehicle routing and scheduling problem. *European Journal of Operational Research*, Elsevier, v. 100, n. 1, p. 180–191, 1997.

BRANDAO, J.; MERCER, A. The multi-trip vehicle routing problem. *Journal of the Operational Research Society*, Taylor & Francis, v. 49, n. 8, p. 799–805, 1998.

BRÄYSY, O. et al. A survey of heuristics for the vehicle routing problem part i: Basic problems and supply side extensions. *SINTEF*, 2008.

BRITO, M. P.; DEKKER, R. *Reverse logistics-a framework*. Burgemeester Oudlaan 50, 3062 PA Rotterdam, 2002.

BRUNDTLAND, G. Our common future: Report of the 1987 world commission on environment and development, vol. 1. *United Nations, Oslo*, p. 59, 1987.

CARUSO, C.; COLORNI, A.; PARUCCINI, M. The regional urban solid waste management system: A modelling approach. *European Journal of Operational Research*, Elsevier, v. 70, n. 1, p. 16–30, 1993.

CATTARUZZA, D.; ABSI, N.; FEILLET, D. Vehicle routing problems with multiple trips. *4OR*, Springer, v. 14, n. 3, p. 223–259, 2016.

CET. *Restricted Zones Operation*. 2021. Accessed: 2021-06-07. Available at: ⟨http://www.cetsp.com.br/consultas/acesso-a-informacao.aspx⟩.

CHAO, I.-M.; GOLDEN, B.; WASIL, E. A computational study of a new heuristic for the site-dependent vehicle routing problem. *INFOR: Information Systems and Operational Research*, Taylor & Francis, v. 37, n. 3, p. 319–336, 1999.

CHAO, I.-M.; LIOU, T.-S. A new tabu search heuristic for the site-dependent vehicle routing problem. In: *The next wave in computing, optimization, and decision technologies*. [S.l.]: Springer, 2005. p. 107–119.

CHRISTOFIDES, N.; MINGOZZI, A.; TOTH, P. Exact algorithms for the vehicle routing problem, based on spanning tree and shortest path relaxations. *Mathematical Programming*, Springer, v. 20, n. 1, p. 255–282, 1981.

CLARKE, G.; WRIGHT, J. W. Scheduling of vehicles from a central depot to a number of delivery points. *Operations Research*, Informs, v. 12, n. 4, p. 568–581, 1964.

COMMISSION, E. *Circular economy action plan*. 2020. Accessed: 2021-06-10. Available at: ⟨https://ec.europa.eu/environment/strategy⟩.

CONAMA. *Resolução CONAMA nº 362 de 23/06/2005*. 2021. Accessed: 2021-06-01. Available at: ⟨https://www.legisweb.com.br/legislacao/?id=102246⟩.

CORDEAU, J.-F. et al. Chapter 7: Vrp with time windows. In: *The Vehicle Routing Problem*. [S.l.]: SIAM, 2002. p. 157–193.

CORDEAU, J.-F.; GENDREAU, M.; LAPORTE, G. A tabu search heuristic for periodic and multi-depot vehicle routing problems. *Networks: An International Journal*, Wiley Online Library, v. 30, n. 2, p. 105–119, 1997.

CORDEAU, J.-F.; LAPORTE, G. A tabu search algorithm for the site dependent vehicle routing problem with time windows. *INFOR: Information Systems and Operational Research*, Taylor & Francis, v. 39, n. 3, p. 292–298, 2001.

DESAULNIERS, G.; LESSARD, F.; HADJAR, A. Tabu search, generalized k-path inequalities, and partial elementarity for the vehicle routing problem with time windows. *Les Cahiers du GERAD ISSN*, v. 711, p. 2440, 2006.

DESAULNIERS, G.; MADSEN, O. B.; ROPKE, S. Chapter 5: The vehicle routing problem with time windows. In: *Vehicle Routing: Problems, Methods, and Applications, Second Edition*. [S.l.]: SIAM, 2014. p. 119–159.

DESROCHERS, M.; DESROSIERS, J.; SOLOMON, M. A new optimization algorithm for the vehicle routing problem with time windows. *Operations Research*, INFORMS, v. 40, n. 2, p. 342–354, 1992.

DOERNER, K. F.; SCHMID, V. Survey: Matheuristics for rich vehicle routing problems. In: SPRINGER. *International Workshop on Hybrid Metaheuristics*. [S.l.], 2010. p. 206–221.

DREXL, M. Rich vehicle routing in theory and practice. *Logistics Research*, Springer, v. 5, n. 1, p. 47–63, 2012.

EUCHI, J.; CHABCHOUB, H. A hybrid tabu search to solve the heterogeneous fixed fleet vehicle routing problem. *Logistics Research*, Springer, v. 2, n. 1, p. 3–11, 2010.

FLEISCHMANN, B. The vehicle routing problem with multiple use of vehicles. *Research report Department of Economics, University "a t Hamburg*, 1990.

FOUNDATION, E. M. Towards the circular economy. *Journal of Industrial Ecology*, v. 2, p. 23–44, 2013.

GIANNIKOS, I. A multiobjective programming model for locating treatment sites and routing hazardous wastes. *European Journal of Operational Research*, Elsevier, v. 104, n. 2, p. 333–342, 1998.

GOOGLE. *Google Maps*. 2021. Accessed: 2021-11-26. Available at: ⟨https://www.google.com.br/maps⟩.

GOLDEN, B. et al. The fleet size and mix vehicle routing problem. *Computers & Operations Research*, Elsevier, v. 11, n. 1, p. 49–66, 1984.

GOLDEN, B. L.; MAGNANTI, T. L.; NGUYEN, H. Q. Implementing vehicle routing algorithms. *Networks*, Wiley Online Library, v. 7, n. 2, p. 113–148, 1977.

GOLDEN, B. L.; RAGHAVAN, S.; WASIL, E. A. *The vehicle routing problem: latest advances and new challenges*. [S.l.]: Springer Science & Business Media, 2008. v. 43.

GROSSO, R. et al. Mathematical formulation and comparison of solution approaches for the vehicle routing problem with access time windows. *Complexity*, Hindawi, v. 2018, 2018.

GUROBI. *Mixed-Integer Programming (MIP) – A Primer on the Basics*. 2021. Accessed: 05/10/2021. Available at: ⟨https://www.gurobi.com/resource/mip-basics/⟩.

HU, T.-L.; SHEU, J.-B.; HUANG, K.-H. A reverse logistics cost minimization model for the treatment of hazardous wastes. *Transportation Research Part E: Logistics and Transportation Review*, Elsevier, v. 38, n. 6, p. 457–473, 2002.

IFORS. *Operations Research Definition by IFORS*. 2021. Accessed: 2021-06-01. Available at: ⟨https://www.ifors.org/what-is-or/⟩.

JEPSEN, M. et al. Subset-row inequalities applied to the vehicle-routing problem with time windows. *Operations Research*, INFORMS, v. 56, n. 2, p. 497–511, 2008.

KOÇ, Ç. et al. Thirty years of heterogeneous vehicle routing. *European Journal of Operational Research*, Elsevier, v. 249, n. 1, p. 1–21, 2016.

LAHYANI, R.; KHEMAKHEM, M.; SEMET, F. Rich vehicle routing problems: From a taxonomy to a definition. *European Journal of Operational Research*, Elsevier, v. 241, n. 1, p. 1–14, 2015.

LAPORTE, G. Fifty years of vehicle routing. *Transportation Science*, INFORMS, v. 43, n. 4, p. 408–416, 2009.

LAPORTE, G.; NOBERT, Y. Exact algorithms for the vehicle routing problem. In: *North-Holland Mathematics Studies*. [S.l.]: Elsevier, 1987. v. 132, p. 147–184.

LAPORTE, G.; SEMET, F. Classical heuristics for the capacitated vrp. In: *The vehicle routing problem*. [S.l.]: SIAM, 2002. p. 109–128.

LENSTRA, J. K.; KAN, A. R. Complexity of vehicle routing and scheduling problems. *Networks*, Wiley Online Library, v. 11, n. 2, p. 221–227, 1981.

LI, F.; GOLDEN, B.; WASIL, E. A record-to-record travel algorithm for solving the heterogeneous fleet vehicle routing problem. *Computers & Operations Research*, Elsevier, v. 34, n. 9, p. 2734–2742, 2007.

LUBEMONITRIX. *Functions of a Lubricant*. 2021. Accessed: 2021-03-18. Available at: ⟨https://www.lubemonitrix.ie/functions-lubricant/⟩.

MANCINI, S. A matheuristic for the vrp with limited traffic zones. *Matheuristics 2016*, p. 35, 2016.

MMA. *Relatório em Atendimento à Resolução CONAMA Nº 362/2005 sobre Óleos Lubrificantes Usados e/ou Contaminados (OLUC)*. R. Vitória - Conj. Hab. Fazenda do Carmo, São Paulo - SP, 2020.

MOURÃO, M. C.; AMADO, L. Heuristic method for a mixed capacitated arc routing problem: A refuse collection application. *European Journal of Operational Research*, Elsevier, v. 160, n. 1, p. 139–153, 2005.

MPRS. *Divulgação da Operação Petrolato*. 2019. Accessed: 2021-03-18. Available at: ⟨https://www.mprs.mp.br/noticias/ambiente/50390⟩.

MURPHY, P. R.; POIST, R. Management of logistical retromovements: an empirical analysis of literature suggestions. In: *Journal of the Transportation Research Forum*. [S.l.: s.n.], 1988. v. 29, n. HS-040 801.

NAG, B. *Vehicle routing with site dependencies. In" Vehicle routing: methods and studies"(BL Golden and AA Assad, eds.)*. [S.l.]: Amsterdam: North-Holland, 1988.

NAGATA, Y.; BRÄYSY, O.; DULLAERT, W. A penalty-based edge assembly memetic algorithm for the vehicle routing problem with time windows. *Computers & Operations Research*, Elsevier, v. 37, n. 4, p. 724–737, 2010.

NETWORKX. *NetworkX - Network Analysis in Python*. 2021. Accessed: 2021-11-26. Available at: ⟨https://networkx.org/⟩.

PINHEIRO, C. T.; QUINA, M. J.; GANDO-FERREIRA, L. M. Management of waste lubricant oil in europe: A circular economy approach. *Critical Reviews in Environmental Science and Technology*, Taylor & Francis, p. 1–36, 2020.

PYTHON. *Python 3*. 2021. Accessed: 2021-11-26. Available at: ⟨https://www.python.org/⟩.

SIMPLIROUTE. *SimpliRoute platform*. 2021. Accessed: 05/10/2021. Available at: ⟨https://www.simpliroute.com/⟩.

SOLOMON, M. M. Algorithms for the vehicle routing and scheduling problems with time window constraints. *Operations Research*, Informs, v. 35, n. 2, p. 254–265, 1987.

SRIVASTAVA, S. K. Green supply-chain management: a state-of-the-art literature review. *International Journal of Management Reviews*, Wiley Online Library, v. 9, n. 1, p. 53–80, 2007.

STAHEL, W. R. The circular economy. *Nature News*, v. 531, n. 7595, p. 435, 2016.

STAHL, H.; MERZ, C. Study to support the commission in gathering structured information and defining of reporting obligations on waste oils and other hazardous waste. *Publications Office of the European Union*, Germany, 2020.

STOCK, J. R. Development and implementation of reverse logistics programs. In: *ANNUAL CONFERENCE PROCEEDINGS, COUNCIL OF LOGISTICS MANAGEMENT.–*. [S.l.: s.n.], 1998.

TAILLARD, É. D. A heuristic column generation method for the heterogeneous fleet vrp. *RAIRO - Operations Research*, v. 33, n. 1, p. 1–14, 1999.

TAILLARD, É. D.; LAPORTE, G.; GENDREAU, M. Vehicle routeing with multiple use of vehicles. *Journal of the Operational Research Society*, Taylor & Francis, v. 47, n. 8, p. 1065–1070, 1996.

TARANTILIS, C. D.; KIRANOUDIS, C. T.; VASSILIADIS, V. S. A threshold accepting metaheuristic for the heterogeneous fixed fleet vehicle routing problem. *European Journal of Operational Research*, Elsevier, v. 152, n. 1, p. 148–158, 2004.

TOTH, P.; VIGO, D. *Vehicle routing: problems, methods, and applications*. [S.l.]: SIAM, 2014.

VIDAL, T. et al. Heuristics for multi-attribute vehicle routing problems: A survey and synthesis. *European Journal of Operational Research*, Elsevier, v. 231, n. 1, p. 1–21, 2013.

VIDAL, T.; LAPORTE, G.; MATL, P. A concise guide to existing and emerging vehicle routing problem variants. *European Journal of Operational Research*, Elsevier, v. 286, n. 2, p. 401–416, 2020.

WINSTON, W. L.; GOLDBERG, J. B. *Operations research: applications and algorithms*. [S.l.]: Thomson/Brooks/Cole Belmontˆ eCalif Calif, 2004. v. 3.